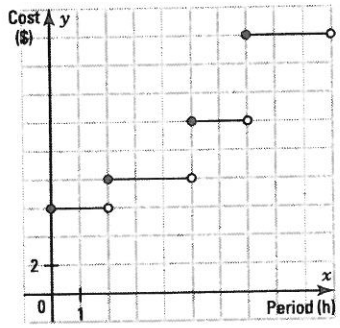
2.8 Step function

ACTIVITY 1 Recreation centre

The graph on the right shows, depending on the length of the period, the cost of using a recreation centre that is open from 10 am to 8 pm.



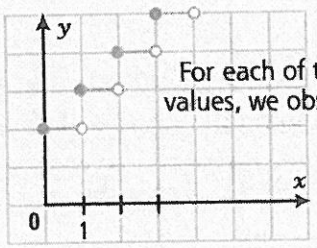
a) What is the cost for a period of
1. 1 h 30? ***$6*** 2. 4 hours? ***$8***
3. 6 hours? ***$12*** 4. 8 hours? ***$18***

b) What is the cost if the length of the period x verifies: $5 \leq x < 7$? ***$12***

c) If the cost is \$8, in what interval lies the length x of the period?
$x \in [2, 5[$

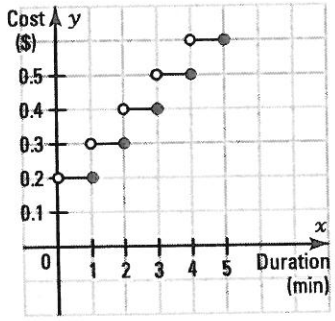
STEP FUNCTION

The graph of a step function is composed of horizontal segments (steps), usually open at one end and closed at the other.
- step closed on the left and open on the right: ●—○
- step open on the left and closed on the right: ○—●



For each of the critical values, we observe a jump.

1. A telephone company charges for long-distance calls in North America in the following way: \$0.20 for the first minute or fraction of the first minute, plus \$0.10 per additional minute or fraction of an additional minute.



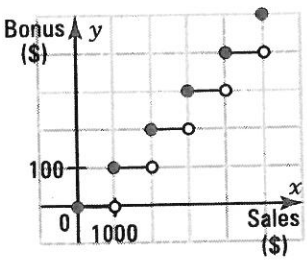
a) What is the cost of a call whose duration is
1. 2 min? ***$0.30*** 2. 2.5 min? ***$0.40*** 3. 3 min? ***$0.40***

b) Draw the Cartesian graph of the function f which associates the duration x, in minutes, of a call with its cost y in dollars.

c) Complete using the appropriate term "open" or "closed".
Each step is ***open*** on the left and ***closed*** on the right.

d) What is the duration of a call that cost \$0.50? ***$x \in]3, 4]$***

2. To motivate his sales staff, the sales manager offers a \$100 bonus for each \$1000 amount of sales, up to a maximum of \$5000 in sales.



a) Draw the Cartesian graph of the function which associates the amount x of sales with the bonus y received by a salesperson.

b) What is the bonus for a sales amount of \$3500? ***$300***

c) In which interval lies the sales amount if the bonus is \$400?
$[4000, 5000[$

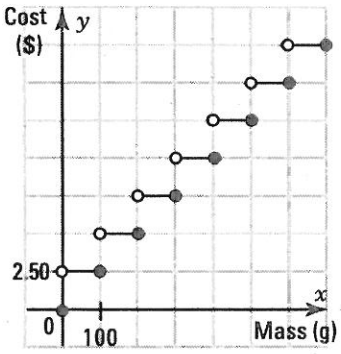
3. A salesman receives a \$50 bonus for each \$1000 in sales made in one week.

a) A salesman makes \$12 480 in sales during a week. What will his salary be? ***\$600***

b) For what amount of sales will the salesman receive a salary equal to \$1000?
For an amount in [20 000, 21 000[.

c) Is it possible for the salesman to receive a salary equal to \$825? Justify your answer.
No, since 825 is not a multiple of 50.

4. The cost y, in dollars, of sending a parcel depends on its mass x, in grams. This cost is represented on the right.

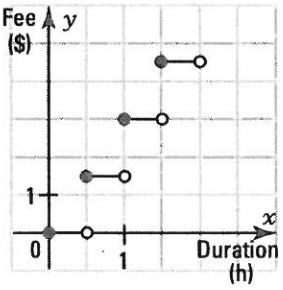


a) What is the cost of sending a parcel whose mass is 260 g?
\$7.50

b) In what interval is the mass of a parcel that cost \$12.50 to send?
$x \in$]400, 500]

c) Explain, using words, how to calculate the cost of sending a parcel. ***We must pay \$2.50 for 100 g or less plus \$2.50 for each additional 100 g.***

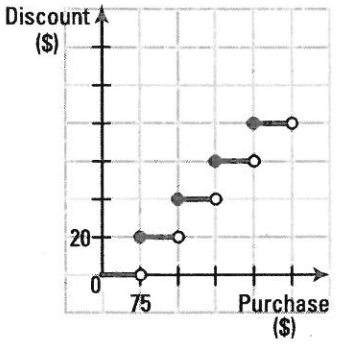
5. In a parking lot, the fee y is calculated in the following way: a cost of \$1.50 for each incomplete 30-minute period. A period of less than 30 minutes is free.



a) Draw the graph of the function which associates the parking duration x, in hours, with the fee y, in dollars.

b) What parking duration corresponds to a fee of \$9?
$x \in$ [3; 3.5[

6. An electronic store announces, during winter sales, a discount of \$20 for every purchase of \$75, without taxes. Karen purchases an electronic tablet and receives a discount of \$60. Valerie also purchases the same tablet and a video game costing \$128 and receives a discount of \$80.



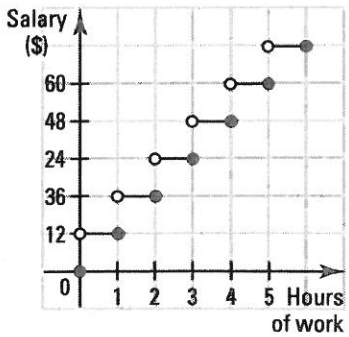
What are the possible prices, without taxes, of the electronic tablet that Karen and Valerie both purchased?

K : $x \in$ [225, 300[

V : $(x + 128) \in$ [300, 375[$\Leftrightarrow x \in$ [172, 247[

The cost of the tablet belongs to the interval [225, 247[

7. Nathalie works weekends in a sports center. She received \$12 for every hour of work, whether the hour is completed or not. Sunday, Nathalie worked twice as long as Saturday. Will the salary Nathalie earned Sunday be double the salary earned Saturday? Justify your answer.



No. Counter example: If Nathalie worked 2h30 min on Saturday, she would receive a salary of \$36. If Nathalie worked 5h on Sunday, she would receive a salary of \$60.

Evaluation 2

1. Find the rule of the following functions.

a) f is a constant function such that $f(1) = 2$. $f(x) = 2$

b) f is linear function of direct variation such that $f(1) = 2$. $f(x) = 2x$

c) f is a linear function such that $f(1) = 2$ and $f(3) = 5$. $f(x) = \frac{3}{2}x + \frac{1}{2}$

d) f is a function for which the product of the variables x and y is equal to 12. $f(x) = \frac{12}{x}$

e) f is a quadratic function whose graph has vertex V(0, 0) and such that $f(2) = 6$.
$f(x) = \frac{3}{2}x^2$

f) f is an exponential function whose graph passes through the points (0, 5) and (2, 20)
$f(x) = 5(2)^x$

2. For each of the following tables of values, find the rule of the function associated with it.

a)

x	0	1	2	3
y	16	8	4	2

$y = 16\left(\frac{1}{2}\right)^x$

b)

x	0	1	2	3
y	–2	–2	–2	–2

$y = -2$

c)

x	1	2	3	4
y	12	6	4	3

$y = \frac{12}{x}$

d)

x	0	1	2	3
y	0	0.25	1	2.25

$y = \frac{1}{4}x^2$

e)

x	0	1	2	3
y	8	12	18	27

$y = 8\left(\frac{3}{2}\right)^x$

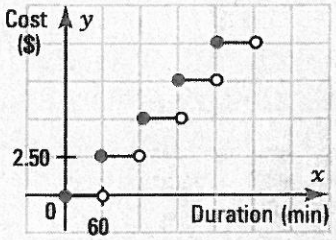
f)

x	2	4	6	8
y	1	4	7	10

$y = \frac{3}{2}x - 2$

3. A the beginning of a car trip, the gas tank contains 66 litres. After traveling 50 km, the tank contains 60 litres. What is the rule of the linear function which associates the distance x traveled, in kilometres, with the quantity y, in litres, of gas left in the tank? $y = 66 - 0.12x$

4. The cost of parking in a garage is represented in the Cartesian plane on the right. Consider the function which associates the parking duration, in minutes, with the cost y in dollars.

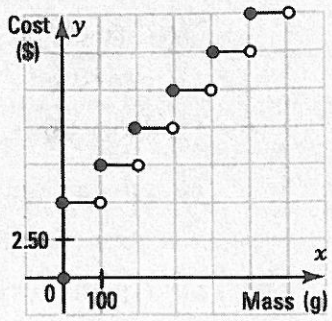


a) Describe, using words, how the cost is calculated.
Parking is free for a duration less than 60 minutes. There are charges of $2.50 for each additional complete 60-minute period.

b) What is the cost for 2 h 15 min? $5

c) What is the possible duration if we pay $12.50? $300 \leq x < 360$

5. The cost (in \$) of sending a parcel depends on its mass (in g). The cost is \$5 for a mass less than 100 g and \$2.50 for each additional 100 g.

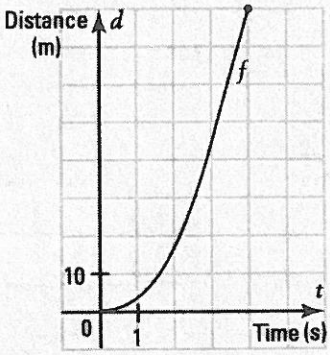


a) Draw the graph of the function which associates the mass x of the parcel with the cost y of sending it.

b) What is the cost of sending a 325 g parcel? ***\$12.50***

c) In what interval lies the mass of a parcel if it costs \$15 to send it? ***[400, 500[***

6. From the top of 80 m tall building, an object is thrown vertically downward. The function f which associates the time t (in s) elapsed since the start with the distance d traveled (in m) has the rule: $d = 5t^2$.



a) Represent function f in the Cartesian plane on the right.

b) At what time t does the object hit the ground? ***4 s***

c) Determine in this situation

1. dom f. ***[0, 4]*** 2. ran f. ***[0, 80]***

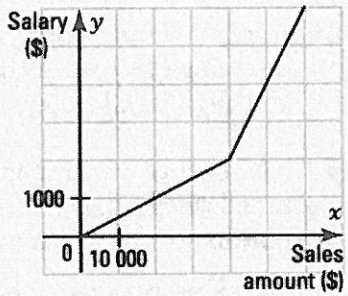
7. A herd presently contains 7 elephants. This herd doubles every 6 years. After how many years will the herd contain 112 elephants? ***After 24 years***

8. A capital of \$1000 is invested during 5 years at an interest rate of 10% compounded annually. Determine the accumulated capital. ***$y = 1000(1.10)^5 = \$1610.51$***

9. A ball bounces to a height equal to $\frac{3}{5}$ of the height reached with the previous rebound. The ball is dropped from a 25 m tall building. What height does the ball reach after the sixth rebound? ***1.17 m***

10. The monthly salary y of an employee depends on the amount of sales made during the month. The function f which gives the employee's salary has rule:

$$f(x) = \begin{cases} 0.05x & 0 \le x < 40\,000 \\ 0.2x - 6000 & x \ge 40\,000 \end{cases}$$



a) Represent the function in the Cartesian plane on the right.

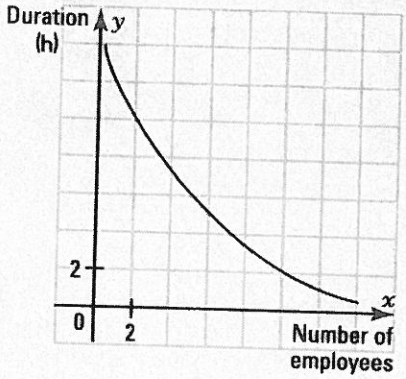
b) What is the salary of an employee who makes \$30 000 in sales in a month? ***\$1500***

c) What is the amount of sales made by an employee who receives a salary of \$4700? ***\$53 500***

11. A company wishes to do renovation work in a building in order to reduce energy costs. The renovations require a total of 20 hours.

a) Draw the graph of the function f which associates the number x of employees with the duration y of the renovations.

b) What is the rule of this function? $y = \frac{20}{x}$



12. Aurelia and Raphael purchase the same day a car at a dealership. The value of Aurelia's car depreciates by 20% per year and the value of Raphael's car depreciates by 14% per year. Aurelia purchased her car for \$24 000 and Raphael purchased his for \$28 500.

What will be the value of Raphael's car at the time that Aurelia's car is worth \$15 360?

Aurelia: $y = 24\,000(0.8)^t$; $15\,360 = 24\,000(0.8)^t$; $0.8^t = 0.64$; $t = 2$

Raphael: $y = 28\,500(0.86)^2 = \$21\,078.60$

13. An epidemic reaches a village in Africa. At the beginning of the epidemic, 400 infected people are counted. Every month, this number increases by 12% compared to the previous month. Six months after the beginning of the epidemic, it ceases to spread and the infected population decreases by 20% every 4 months.

What is the number of people infected 16 months after the beginning of the epidemic?

After 6 months: $y = 400(1.12)^t$; $y = 400(1.12)^6 = 790$ *infected people*

After 16 months: $y = 790(0.8)^{\frac{1}{4}t} = 790(0.8)^{\frac{1}{4}(12)} = 404$ *infected people*

14. Sylvia is a seller in a store of household appliances. She receives a commission of \$120 for every \$1000 of sales made.
In January, she received a \$360 commission. If, in February, she sells twice as much as in January, will she receive double the commission? Justify your answer.

No

January: amount of sales $\in [3\,000, 4\,000[$

For exemple: \$3800 in sales

Commission: \$360

February: amount of sales \$7600 (2 x \$3800)

Commission: \$840