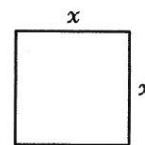


2.5 Quadratic function

ACTIVITY 1 Area of a square

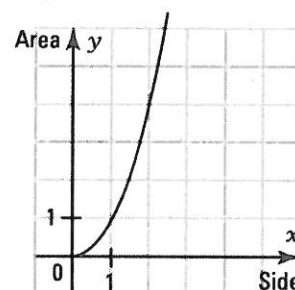
Consider the square with side length x shown on the right.

- a) What is the rule of the function which associates the side x with the area y of this square? $y = x^2$



- b) Complete the table of values giving the area y as a function of the side length x .

x	0	0.5	1	1.5	2	3
y	0	0.25	1	2.25	4	9



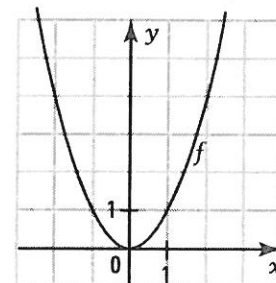
- c) Represent the function in the Cartesian plane.
- d) Explain why the domain of the function is \mathbb{R}_+ .
The side x of a square cannot be negative.
- e) Is the rate of change between any two points on the graph constant?
No

ACTIVITY 2 Basic quadratic function

Consider the function $f(x) = x^2$.

- a) Complete the table of values below.

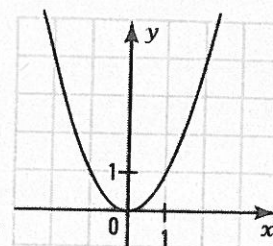
x	-2	-1	0	1	2
y	4	1	0	1	4



- b) Represent the function in the Cartesian plane.
- c) 1. Explain why $f(-x) = f(x)$, for all x .
Two opposite numbers have the same square. Indeed, $(-x)^2 = x^2$.
2. Therefore, what does the y -axis represent for the curve drawn? *An axis of symmetry.*
- d) Determine
- | | | | |
|----------------------|--------------|--------------------------------|----------------|
| 1. dom f . | \mathbb{R} | 2. ran f . | \mathbb{R}_+ |
| 3. the zero of f . | 0 | 4. the y -intercept of f . | 0 |
- e) What is the sign of function f ? $f(x) \geq 0$ over \mathbb{R}
- f) Over what interval is function f
- | | | | |
|----------------|----------------|----------------|----------------|
| 1. decreasing? | $]-\infty, 0]$ | 2. increasing? | $[0, +\infty[$ |
|----------------|----------------|----------------|----------------|
- g) What is the minimum of function f ? 0

BASIC QUADRATIC FUNCTION

- The function $f(x) = x^2$ is called the basic quadratic function.
- The Cartesian graph is a parabola with vertex $V(0, 0)$.
 - $\text{dom } f = \mathbb{R}$.
 - $\text{ran } f = \mathbb{R}_+$.
 - The y-intercept of f is 0.
 - The function has only one zero, which is equal to 0. $\forall x \in \mathbb{R}: f(x) \geq 0$.
 - The function is decreasing over $]-\infty, 0]$, increasing over $[0, +\infty[$.
 - The minimum of the function is 0.
 - The rate of change between any two points on the graph is not constant.
 - The y-axis with equation $x = 0$ is an axis of symmetry for the parabola. $\forall x \in \mathbb{R}: f(-x) = f(x)$.



1. Consider the basic quadratic function $f(x) = x^2$.

a) Explain how to deduce the graph of the function $g(x) = -x^2$

By a reflection about the x-axis.

b) Draw the graph of function g .

c) Determine

1. $\text{dom } g$. \mathbb{R} 2. $\text{ran } g$. \mathbb{R}_-

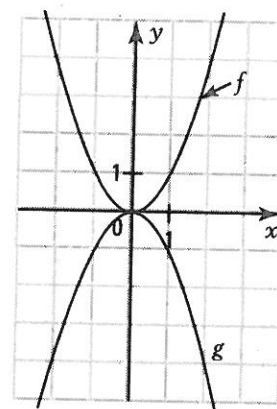
3. the zero of g . 0 4. the y-intercept of g . 0

d) What is the sign of function g ? $g(x) \leq 0, \forall x \in \mathbb{R}$

e) Over what interval is function g

1. increasing? $]-\infty, 0]$ 2. decreasing? $[0, +\infty[$

f) What is the maximum of function g ? 0



ACTIVITY 3 Role of parameter a

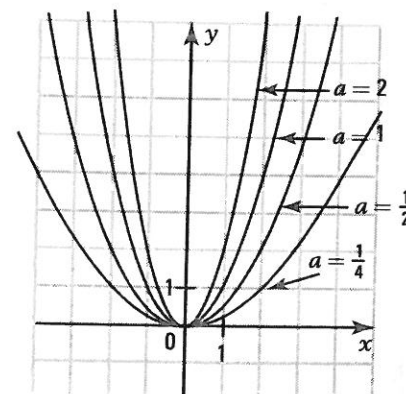
a) Consider the basic quadratic function $y = x^2$ and the function $f(x) = ax^2$ ($a > 0$).

1. Represent function f when

1) $a = \frac{1}{4}$. 2) $a = \frac{1}{2}$. 3) $a = 2$.

2. As parameter a increases, do you observe a vertical stretch or reduction of the parabola?

A vertical stretch.



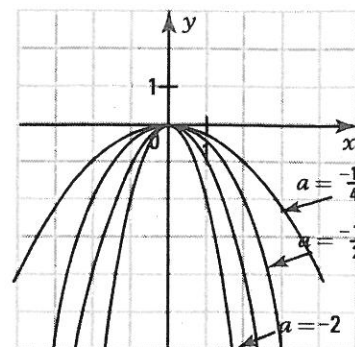
- b) Consider the quadratic function $y = -x^2$ and the function $f(x) = ax^2$ ($a < 0$).

1. Represent function f when

1) $a = -\frac{1}{4}$ 2) $a = -\frac{1}{2}$ 3) $a = -2$.

2. As the absolute value of parameter a increases, do you observe a vertical stretch or reduction?

A vertical stretch.



- c) Consider the parabola with equation $y = ax^2$.

Is the parabola open upward or downward when

1. $a > 0$? **Upward** 2. $a < 0$? **Downward**

ACTIVITY 4 Interpretation of parameter a

A real estate agent sells square-shaped lots. The cost of each lot is \$10 per m^2 .

- a) Find the rule of the function f which associates the measure x of the side with the cost y of the lot. **$y = 10x^2$**

- b) The rule of the function is of the form $y = ax^2$.

1. Identify parameter a . **$a = 10$**

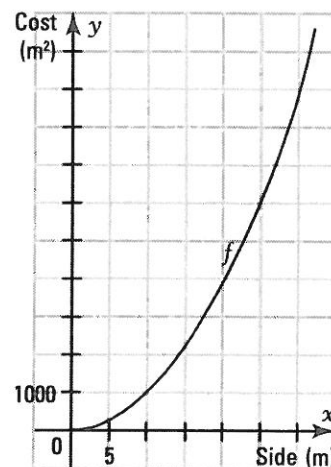
2. Interpret parameter a in this situation.

a represents the cost per m^2 of each lot.

We say that the cost y is **directly proportional** to the square of the side length x .

- c) Complete the table of values of function f .

x	0	10	15	20	30
y	0	1000	2250	4000	9000

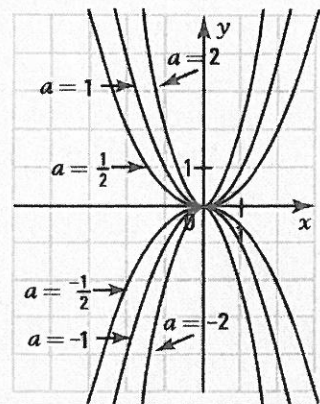


- d) Represent function f in the Cartesian plane on the right.

ROLE OF PARAMETER a

Consider the parabola with equation $y = ax^2$ and vertex $V(0, 0)$.

- The **sign** of a determines whether the parabola is open upward or downward.
 - $a > 0$: the parabola is open upward.
 - $a < 0$: the parabola is open downward.
- The **absolute value** of a influences the opening of the parabola. As the absolute value of a increases, we observe a **vertical stretch** of the parabola.



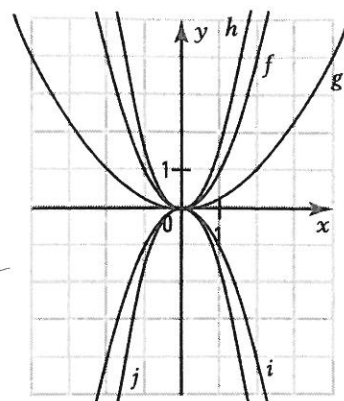
2. The graph of the function $f(x) = x^2$ is drawn on the right. Deduce the graph of

a) $g(x) = \frac{1}{4}x^2$.

b) $h(x) = \frac{3}{2}x^2$.

c) $i(x) = -x^2$.

d) $j(x) = -2x^2$.



ACTIVITY 5 Graphing a parabola

Consider the function $f(x) = -2x^2$.

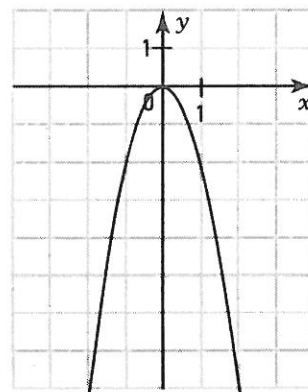
- a) Is the parabola open upward or downward? Justify your answer.
Downward since $a < 0$.

- b) Determine the coordinates of the vertex. **$V(0, 0)$**

- c) Complete the table of values below.

x	-2	-1	0	1	2
$y = -2x^2$	-8	-2	0	-2	-8

- d) Draw the parabola in the Cartesian plane on the right.



GRAPHING THE PARABOLA $y = ax^2$

Procedure

- Determine the opening according to the sign of a .
- Determine the coordinates of the vertex.
- Complete a table of values.
- Draw the parabola.

Ex.: $y = \frac{1}{2}x^2$

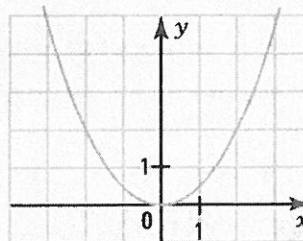
1. Open upward since $a > 0$.

2. $V(0, 0)$

- 3.

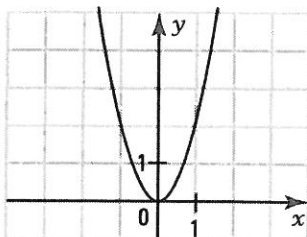
x	-3	-2	-1	0	1	2	3
$y = \frac{1}{2}x^2$	4.5	2	0.5	0	0.5	2	4.5

- 4.

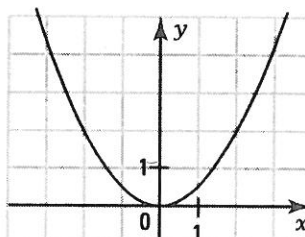


3. Draw the following parabolas.

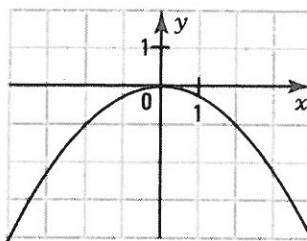
a) $y = 2x^2$



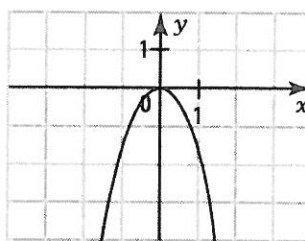
b) $y = \frac{1}{2}x^2$



c) $y = -\frac{1}{4}x^2$



d) $y = -\frac{3}{2}x^2$



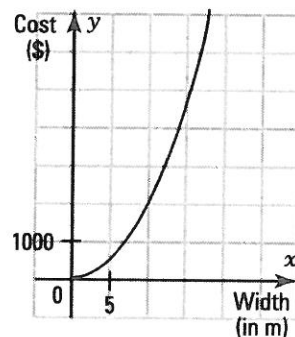
4. A rectangular field is twice as long as it is wide. Land is sold \$10/m².

- a) Find the rule of the function which associates the width x (in m) of the field with its cost C (in \$).

$C = 20x^2$

- b) Draw the graph of this function.

- c) If the field cost \$8000, what will be the cost of putting a fence around it if each metre of fence costs \$30? $\$3600$



5. A square-based prism has a height equal to 25 cm. Let x represent the length of the base's edge.

- a) Find the rule of the function which associates the length of the base's edge with the volume V of the prism. $V = 25x^2$

- b) If the prism has its volume equal to 3600 cm³, what is the length of the base's edge? 12 cm .

6. The distance d (in m) traveled by a free-falling object as a function of the time t (in sec) elapsed since it was dropped is described by the rule $d = 4.9 t^2$.

- a) Complete the table of values on the right.

t	0	1	2	3	4
d	0	4.9	19.6	44.1	78.4

- b) Calculate the average speed of the object between time

1. 0 s and 1 s. $\frac{4.9}{1} \text{ m/s}$ 2. 1 s and 2 s. $\frac{14.7}{1} \text{ m/s}$ 3. 2 s and 3 s. $\frac{24.5}{1} \text{ m/s}$ 4. 3 s and 4 s. $\frac{34.3}{1} \text{ m/s}$

- c) Is the average speed of the object constant? What can you say about it?

The average speed is not constant. It increases with time.

- d) After how many seconds, rounded to the nearest unit, will the object hit the ground if it was dropped from a height of 180 m? After about 6 seconds.

- e) The speed v (in m/s) of the object at time t is described by the rule $v = 9.8t$. Determine, rounded to the nearest unit, the speed of the object when it hits the ground.

1. In m/s. 59 m/s 2. In km/h. 212 km/h

7. A real estate agent is selling square-shaped lots. A lot with sides of length 25 m is sold \$11 250.

- a) What is the rule of the function which associates the length x of the side of a lot with its cost C (in \$)? $C = 18x^2$
 b) What is the cost of a lot with sides of length 20 m? \$7200
 c) What is the side length of a lot sold for \$16 200? 30 m

8. A square-based pyramid has a height equal to 12 cm. Let x represent the length (in cm) of the base's edge and V its volume.

- a) Find the rule of the function which associates the edge length x with the volume V . $V = 4x^2$

- b) Find the rule of the function which associates the volume V with the edge length x .

$$x = \frac{\sqrt{V}}{2}$$

- c) Determine

1. the volume of the pyramid if the base's edge measures 10 cm. 400 cm³
 2. the length of the edge of the pyramid's base if its volume is equal to 100 cm³. 5 cm

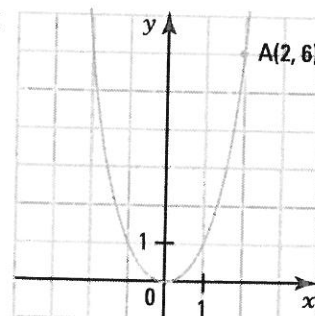
ACTIVITY 6 Finding the rule $y = ax^2$

The parabola on the right has vertex $V(0, 0)$ and passes through the point $A(2, 6)$.

- a) Explain how to find the equation of the parabola. In the equation $y = ax^2$, we find a after replacing x and y with the coordinates of point A.

- b) Find the equation of the parabola.

$$6 = a \cdot 2^2 \Rightarrow a = \frac{3}{2}. \text{ Thus, } y = \frac{3}{2}x^2.$$

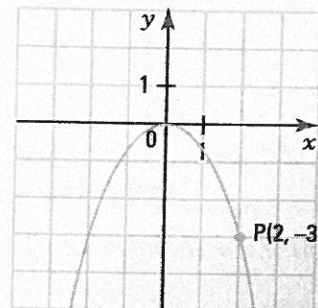


FINDING THE RULE $y = ax^2$

We find the value of parameter a by replacing, in the rule, x and y by the coordinates of the given point P.

$$-3 = a(2)^2 \Rightarrow -3 = 4a \Rightarrow a = -\frac{3}{4}$$

We deduce the rule: $y = -\frac{3}{4}x^2$.



9. Find the equation of the parabola with vertex $V(0,0)$ and passing through the given point P.

- a) $P(1, 2)$ $y = 2x^2$ b) $P(-2, -12)$ $y = -3x^2$
 c) $P(-3, 6)$ $y = \frac{2}{3}x^2$ d) $P(2, -6)$ $y = -\frac{3}{2}x^2$

10. The table of values on the right associates the time t (in s) elapsed since a free-falling object was dropped with the distance d (in m) traveled by the object, rounded to the nearest unit.

$t(s)$	0	1	2
$d(m)$	0	5	20

- a) Find the rule of the function which associates the time elapsed t with the distanced traveled, knowing that the distance traveled is directly proportional to the square of the elapsed time.

$d = 5t^2$

- b) Find the rule of the function which associates the distance d traveled with the elapsed time t . $t = \sqrt{\frac{d}{5}}$

- c) Determine

1. the distance traveled by the object 4 seconds after it was dropped. 80 m
 2. the time required for the object to travel a distance of 180 m. 6 sec

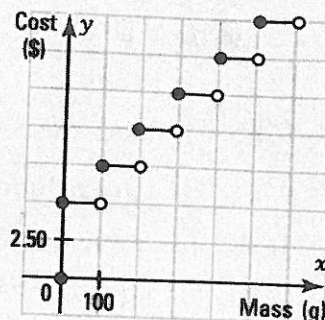
11. Consider the table of values on the right. Find the rule of the function if

x	0	2
y	0	4

- a) y is directly proportional to x . $y = 2x$
 b) y is directly proportional to the square of x . $y = x^2$

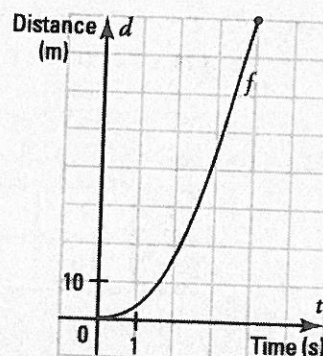
5. The cost (in \$) of sending a parcel depends on its mass (in g). The cost is \$5 for a mass less than 100 g and \$2.50 for each additional 100 g.

- Draw the graph of the function which associates the mass x of the parcel with the cost y of sending it.
- What is the cost of sending a 325 g parcel? \$12.50
- In what interval lies the mass of a parcel if it costs \$15 to send it? [400, 500[



6. From the top of 80 m tall building, an object is thrown vertically downward. The function f which associates the time t (in s) elapsed since the start with the distance d traveled (in m) has the rule: $d = 5t^2$.

- Represent function f in the Cartesian plane on the right.
- At what time t does the object hit the ground? 4 s
- Determine in this situation
 - dom f . [0, 4]
 - ran f . [0, 80]



7. A herd presently contains 7 elephants. This herd doubles every 6 years. After how many years will the herd contain 112 elephants? After 24 years

8. A capital of \$1000 is invested during 5 years at an interest rate of 10% compounded annually. Determine the accumulated capital. $y = 1000(1.10)^5 = \$1610.51$

9. A ball bounces to a height equal to $\frac{3}{5}$ of the height reached with the previous rebound. The ball is dropped from a 25 m tall building. What height does the ball reach after the sixth rebound? 1.17 m

10. The monthly salary y of an employee depends on the amount of sales made during the month. The function f which gives the employee's salary has rule:

$$f(x) = \begin{cases} 0.05x & 0 \leq x < 40\,000 \\ 0.2x - 6000 & x \geq 40\,000 \end{cases}$$

- Represent the function in the Cartesian plane on the right.
- What is the salary of an employee who makes \$30 000 in sales in a month? \$1500
- What is the amount of sales made by an employee who receives a salary of \$4700? \$53 500

