

4.3 Algebraic solving of a two-variable first degree system

ACTIVITY 1 Solving by addition

Consider the system $\begin{cases} 3x + 2y = 7 & (1) \\ 2x - 3y = -4 & (2) \end{cases}$ ← The coefficients of x are not opposite.

- a) By multiplying both sides of the 1st equation by 2 and both sides of the 2nd equation by -3 , we obtain the following system: $\begin{cases} 6x + 4y = 14 & (3) \\ -6x + 9y = 12 & (4) \end{cases}$ ← The coefficients of x are opposite.

Explain why the resulting system is equivalent to the initial system and therefore has the same solution set.

The equations (1) and (3) and the equations (2) and (4) are equivalent.

- b) Add the same sides of the equations of the second system to determine the value of the variable y .

$$13y = 26 \Rightarrow y = 2$$

- c) What is the value of the variable x ? $x = 1$

- d) What is the solution set of the system? $S = \{(1, 2)\}$

ALGEBRAIC SOLVING OF A SYSTEM: ADDITION METHOD

The addition method for solving a system (also called elimination method) is illustrated in the following example.

Given the system $\begin{cases} 2x + 5y = -4 \\ 3x - 2y = 13 \end{cases}$

$$\begin{array}{l} \times 3 \\ \times -2 \end{array} \begin{cases} 2x + 5y = -4 \\ 3x - 2y = 13 \end{cases}$$

$$\begin{cases} 6x + 15y = -12 \\ -6x + 4y = -26 \end{cases}$$

- We multiply the sides of each equation by a non zero real number in order to get opposite coefficients for the variable x (or the variable y).
- We add the same sides of the equations of the resulting system to obtain an equation in only one variable.
- We determine the value of this variable.
- We substitute this value into one of the system's equations to deduce the value of the other variable.
- We establish the solution set S of the system.

$$19y = -38$$

$$y = -2$$

$$2x + 5(-2) = -4$$

$$x = 3$$

$$S = \{(3, -2)\}$$

1. Solve the following systems by addition.

a) $\begin{cases} 3x + 5y = 9 \\ 2x + y = -1 \end{cases}$

$$S = \{(-2, 3)\}$$

b) $\begin{cases} 5x + 3y = -3 \\ 3x + 2y = -1 \end{cases}$

$$S = \{(-3, 4)\}$$

c) $\begin{cases} -3x + 10y = 2 \\ x - 5y = 1 \end{cases}$

$$S = \{(-4, -1)\}$$

$$\text{d) } \begin{cases} x + 6y = 6 \\ x - 4y = 1 \end{cases}$$

$$S = \left\{ \left(3, \frac{1}{2} \right) \right\}$$

$$\text{e) } \begin{cases} 4x + y = -1 \\ 8x + 3y = 0 \end{cases}$$

$$S = \left\{ \left(-\frac{3}{4}, 2 \right) \right\}$$

$$\text{f) } \begin{cases} 3x + 2y = -1 \\ 6x - 4y = 10 \end{cases}$$

$$S = \left\{ \left(\frac{2}{3}, -\frac{3}{2} \right) \right\}$$

2. In each of the following situations,

1. identify the variables.
2. translate the situation into a system of two first degree equations with two variables.
3. solve the system by addition and give a complete answer.

- a) The sum of two numbers is equal to 20 and their difference is equal to 4.
What is the product of these two numbers?

1. x : first number
 y : second number

2.
$$\begin{cases} x + y = 20 \\ x - y = 4 \end{cases}$$

3. $S = \{(12, 8)\}$
The product is equal to 96.

- b) The perimeter of a rectangular yard is equal to 60 m. If we double its length and triple its width, the perimeter is then equal to 144 m. What is the area of the initial yard?

1. x : length of yard
 y : width of yard

2.
$$\begin{cases} 2x + 2y = 60 \\ 4x + 6y = 144 \end{cases}$$

3. $S = \{(18, 12)\}$
The area is equal to 216 m².

- c) Julia buys two sweaters and three pairs of pants for \$220 in a store. Evelyn buys three sweaters and two pairs of pants in the same store for \$230.

How much will Sandra pay for four sweaters and two pairs of pants in this same store?

1. x : price of a sweater
 y : price of a pair of pants

2.
$$\begin{cases} 2x + 3y = 220 \\ 3x + 2y = 230 \end{cases}$$

3. $S = \{(50, 40)\}$
Sacha will pay \$280.

- d) Raphael buys a certain number of 50¢ and 10¢ stamps. If he pays \$6.30 for a total of 19 stamps, how many of each type of stamp did he buy?

1. x : number of 50¢ stamps
 y : number of 10¢ stamps

2.
$$\begin{cases} 50x + 10y = 630 \\ x + y = 19 \end{cases}$$

3. $S = \{(11, 8)\}$
Raphael bought eleven 50¢ stamps and eight 10¢ stamps.

- e) Caroline is working this summer at a grocery store and at a pharmacy. The first week, she earned \$138 working 12 h at the grocery store and 8 h at the pharmacy. The second week, she earned \$142 working 8 h at the grocery store and 12 h at the pharmacy. How much will she earn the third week if she works 10 h at the grocery store and 14 h at the pharmacy? **\$170**

- f) A jar contains a total of 20 red, black and blue marbles. There are two more black marbles than red marbles and four more blue marbles than black marbles. How many marbles of each color are there? **4 red, 6 black and 10 blue**

- g) In a warehouse, there is a total of 32 boxes. There are small 240 dm³ boxes and large 320 dm³ boxes. If the total volume occupied by these boxes is 9280 dm³, how many small boxes are there? **12 small boxes**

ACTIVITY 2 Solving by substitution

Consider the system
$$\begin{cases} y = x + 1 & (1) \\ 2x + 3y = 13 & (2) \end{cases}$$

- a) Replace the variable y in equation (2) by the expression $x + 1$. **$2x + 3(x + 1) = 13$**