3.1 Distance between two points

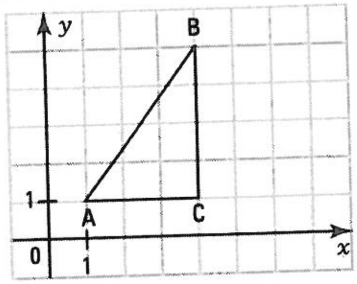
ACTIVITY 1 Distance between two points

Consider the following right triangle with vertices A(1, 1), B(4, 5) and C(4, 1). Find a procedure for calculating the distance between A and B and calculate that distance.

1. Calculate m$\overline{BC}$.   2. Calculate m$\overline{AC}$.

3. Deduce m$\overline{AB}$ using the Pythagorean Theorem.

m$\overline{BC}$ = 4; m$\overline{AC}$ = 3 ⇒ m$\overline{AB}$ = 5



DISTANCE BETWEEN TWO POINTS

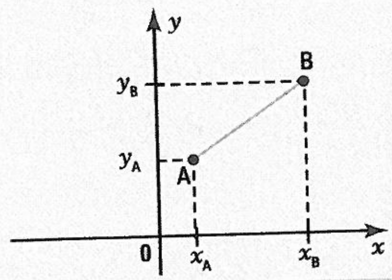
- The distance between two points A(x_A, y_A) and B(x_B, y_B), noted d(A, B), is given by the formula:

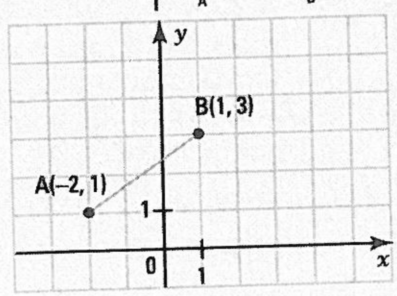
$$\boxed{d(\mathrm{A}, \mathrm{B}) = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}}$$

- A distance is always positive or zero: $d(\mathrm{A}, \mathrm{B}) \geqslant 0$.
- Given two points A and B, we have: $d(\mathrm{A}, \mathrm{B}) = d(\mathrm{B}, \mathrm{A})$.

  Ex.: The distance between A(–2, 1) and B(1, 3) is:

$$\begin{aligned} d(\mathrm{A}, \mathrm{B}) &= \sqrt{(1+2)^2 + (3-1)^2} \\ &= \sqrt{9+4} \\ &= \sqrt{13}. \end{aligned}$$





1. Calculate the distance between the following points:

a) (1, 5) and (2, 3) ***$\sqrt{5}$***

b) (–1, 2) and (1, 2) ***2***

c) (1, –2) and (2, 5) ***$\sqrt{50} = 5\sqrt{2}$***

d) (–1, –3) and (2, 3) ***$\sqrt{45} = 3\sqrt{5}$***

e) (1, 2) and (–2, 1) ***$\sqrt{10}$***

f) (–3, –1) and (2, –3) ***$\sqrt{29}$***

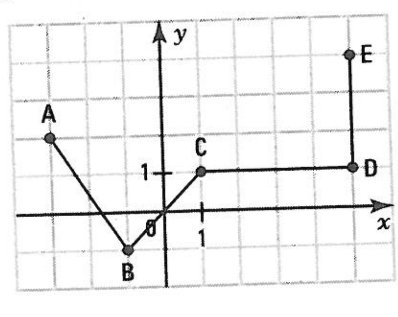
2. What distance separates the following points?

a) A and B ***$\sqrt{13}$***

b) B and C ***$\sqrt{8} = 2\sqrt{2}$***

c) C and D ***4***

d) D and E ***3***

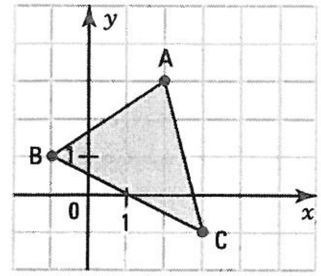


3. Determine the perimeter of triangle ABC on the right.

We have: A(2, 3), B(–1, 1) and C(3, –1); d(A, B) = $\sqrt{13}$

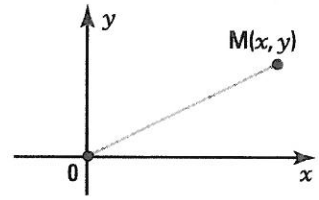
d(B, C) = $\sqrt{20}$ *and d(A, C) =* $\sqrt{17}$ *.*

Perimeter of $\triangle ABC = \sqrt{20} + \sqrt{13} + \sqrt{17} \approx 12.20$ *u*



4. a) Given a random point $M(x, y)$ of the Cartesian plane. Show that $d(0, M) = \sqrt{x^2 + y^2}$.

$d(0, M) = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}$

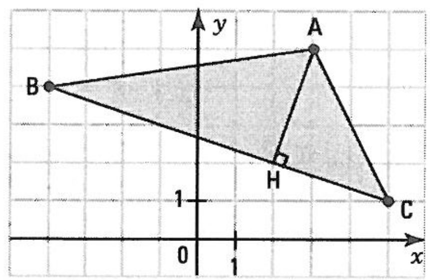


b) Given A(–2, 3), B(1, –2) and C(–3, 4). Calculate

1. $d(0, A)$. $\sqrt{13}$ 2. $d(0, B)$. $\sqrt{5}$ 3. $d(0, C)$. 5

5. Given A(3, 5), B(–4, 4) and C(5, 1), the vertices of triangle ABC on the right, and H(2, 2) the foot of the altitude to side BC, calculate the area of triangle ABC.

$m\overline{BC} = \sqrt{90} = 3\sqrt{10}$, $m\overline{AH} = \sqrt{10}$. *Area* $\triangle$ *ABC = 15* u^2



6. Given three points in the Cartesian plane A(–4, 1), B(1, 6) and C(1, 1).

a) Show that triangle ABC is an isosceles right triangle.

$d(A, B) = m\overline{AB} = \sqrt{50}$, $d(A, C) = m\overline{AC} = 5$; $d(B, C) = m\overline{BC} = 5$

We have: $(m\overline{AB})^2 = (m\overline{AC})^2 + (m\overline{BC})^2$ *and* $m\overline{AC} = m\overline{BC}$.

We deduce that $\triangle$*ABC is an isosceles right triangle with main vertex C.*

b) What is the area of triangle ABC? *12.5* u^2

7. Show that the points A(–2, 2), B(5, –5) and C(4, 2) are located on a circle with centre $\omega(1, -2)$. What is the radius of this circle?

It must be shown that $d(\omega, A) = d(\omega, B) = d(\omega, C)$.

$d(\omega, A) = 5$; $d(\omega, B) = 5$; $d(\omega, C) = 5$. *The circle has a radius of 5 units.*