

- d) In a class of 30 students, there are six more boys than girls. What percentage of this class are girls?

1. x : number of girls
 y : number of boys
 2. $\begin{cases} x + y = 30 \\ y = x + 6 \end{cases}$
 3. $S = \{(12, 18)\}$
 The girls represent 40% of this class.

- e) There are 52 boats in a marina: sailboats and speedboats. There are 3 times as many sailboats as speedboats. How many sailboats and speedboats are there?

39 sailboats and 13 speedboats

- f) The length of a rectangle measures 5 times its width. If the perimeter of this rectangle is 144 cm, what is its area? **720 cm²**

ACTIVITY 3 Solving by comparison

Consider the system $\begin{cases} y = -3x + 5 & (1) \\ y = 2x - 5 & (2) \end{cases}$

- a) What equation with the variable x can we deduce by comparing the two equations of the system obtained in a)? **$-3x + 5 = 2x - 5$**
- b) Solve this last equation to determine the value of the variable x . **$x = 2$**
- c) Deduce the value of the variable y . **$y = -1$**
- d) What is the solution set of the system? **$S = \{(2, -1)\}$**

ALGEBRAIC SOLVING OF A SYSTEM: COMPARISON METHOD

The comparison method for solving a system is illustrated in the following example.

Given the system $\begin{cases} y = 2x + 1 & (1) \\ y = -\frac{3}{2}x + \frac{9}{2} & (2) \end{cases}$

– We deduce by transitivity an equation in only one variable.

$$2x + 1 = -\frac{3}{2}x + \frac{9}{2}$$

– We solve the resulting equation.

$$4x + 2 = -3x + 9$$

$$x = 1$$

– Then, we substitute this value into one of the system's equations and deduce the value of the other variable.

$$y = 2 \times 1 + 1$$

$$y = 3$$

– We establish the solution set S of the system.

$$S = \{(1, 3)\}$$

5. Solve the following systems by comparison.

a) $\begin{cases} y = 2x + 9 \\ y = -3x - 1 \end{cases}$

$S = \{(-2, 5)\}$

b) $\begin{cases} x = 2y + 7 \\ x = -4y - 5 \end{cases}$

$S = \{(3, -2)\}$

c) $\begin{cases} y = \frac{3}{4}x + \frac{1}{2} \\ y = \frac{2}{3}x - 1 \end{cases}$

$S = \{(-18, -13)\}$

6. In each of the following situations,

1. identify the variables.
2. translate the situation into a system of two first degree equations with two variables.
3. solve the system by comparison and give a complete answer.

a) A school principal has the choice of two transportation companies to organize a field trip for the students.

The first company charges a base amount of \$120 plus \$1.50 per student. The second company charges a base amount of \$80 plus \$2 per student. How many students must come for the transportation costs to be the same for both companies?

1. x : number of students y : total cost	2. $\begin{cases} y = 1.5x + 120 \\ y = 2x + 80 \end{cases}$	3. $S = \{(80, 240)\}$ For 80 students, they both charge \$240.
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b) Joseph and Nathan are car salesmen for two different dealerships. Joseph receives a weekly base salary of \$350 and a 0.5% commission on his sales. Nathan receives a base salary of \$100 and a 1% commission on his sales. What must be the amount of sales for Joseph and Nathan to receive the same weekly salary?

1. x : amount of sales (\$) y : salary (\$)	2. $\begin{cases} y = 0.005x + 350 \\ y = 0.01x + 100 \end{cases}$	3. $S = \{(50\ 000, 600)\}$ For \$50 000 in sales, Joseph and Nathan both receive a salary of \$600.
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c) A line l_1 has a slope of $\frac{3}{2}$ and a y -intercept of -3 . A line l_2 , perpendicular to l_1 , has a y -intercept of 10. What is the point of intersection of these two lines?

1. x : x -coordinate of the intersection point y : y -coordinate of the intersection point	2. $\begin{cases} y = \frac{3}{2}x - 3 \\ y = -\frac{2}{3}x + 10 \end{cases}$	3. $S = \{(6, 6)\}$ The intersection point is (6, 6).
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d) Caroline receives a weekly base salary of \$120 plus a \$10 commission for every item sold. Her friend Jessica receives a weekly base salary of \$150 and an \$8 commission for every item sold. How many items must they each sell to earn the same weekly salary?

1. x : number of items sold y : salary (\$)	2. $\begin{cases} y = 10x + 120 \\ y = 8x + 150 \end{cases}$	3. $S = \{(15, 270)\}$ They must each sell 15 items.
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SOLVING A SYSTEM: CHOOSING A METHOD

If a system is written in the form:

- $\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$, we usually solve it by addition
- $\begin{cases} a_1x + b_1y = c_1 \\ y = a_2x + b_2 \end{cases}$, we usually solve it by substitution
- $\begin{cases} y = a_1x + b_1 \\ y = a_2x + b_2 \end{cases}$, we usually solve it by comparison.

7. Solve each of the following systems using the appropriate method.

a) $\begin{cases} y = x - 8 \\ y = -2x + 1 \end{cases}$

$S = \{(3, -5)\}$

b) $\begin{cases} 3x + 2y = -2 \\ 5x + y = 6 \end{cases}$

$S = \{(2, -4)\}$

c) $\begin{cases} y = -2x + 7 \\ 5x - 2y = 4 \end{cases}$

$S = \{(2, 3)\}$