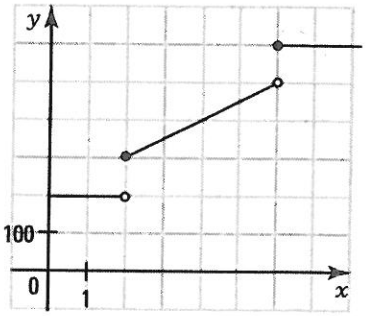
2.7 Piecewise function

ACTIVITY 1 Salary of an employee

The weekly salary $f(x)$ of an employee at a company selling electronic games is calculated according to the number x of games sold in the following way:

$$f(x) = \begin{cases} 200 & \text{if } x < 2 \\ 50x + 200 & \text{if } 2 \leq x < 6 \\ 600 & \text{if } x \geq 6 \end{cases}$$



a) What is the salary of an employee who sells

1. 2 games? ***$300*** 2. 4 games? ***$400*** 3. 12 games? ***$600***

b) Determine the number of games sold by an employee who received a salary of

1. $200. ***0 or 1 game*** 2. $450. ***5 games*** 3. $600. ***6 or more games***

PIECEWISE FUNCTION

A piecewise function is a function where the rule differs according to the interval containing the variable x.

Ex.: Consider the following piecewise function:

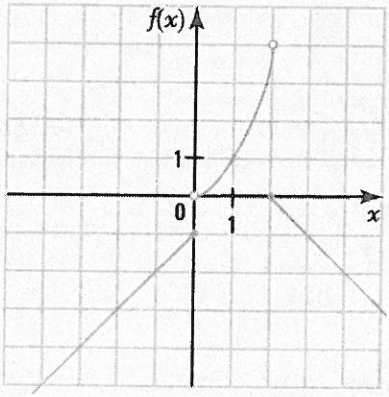
$$f(x) = \begin{cases} x - 1 & \text{if } x \leq 0 \\ x^2 & \text{if } 0 < x < 2 \\ -x + 2 & \text{if } x \geq 2 \end{cases}$$

The graph of this function is represented in the Cartesian plane on the right.

dom $f = \mathbb{R}$, ran $f =]-\infty, 4[$

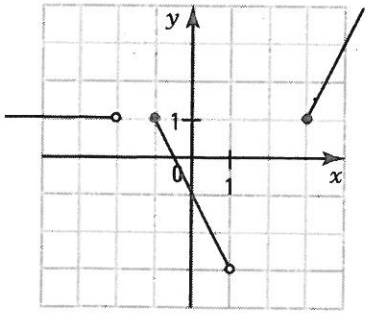
When we evaluate the function for a value of the variable x, we first find to which interval this value belongs and then we use function's rule defined in this interval.

Thus, $f(1.5) = (1.5)^2 = 2.25$; $f(3) = -(3) + 2 = -1$.

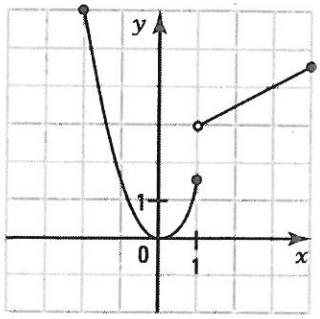


1. Represent the following piecewise functions.

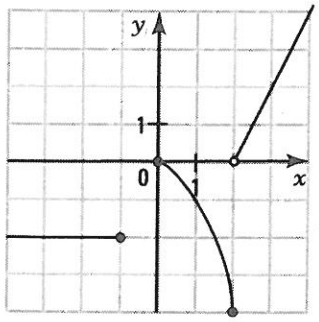
a) $f_1(x) = \begin{cases} 1 & \text{if } x < -2 \\ -2x - 1 & \text{if } -1 \leq x < 1 \\ 2x - 5 & \text{if } x \geq 3 \end{cases}$



b) $f_2(x) = \begin{cases} \frac{3}{2}x^2 & \text{if } -2 \leq x \leq 1 \\ \frac{1}{2}x + \frac{5}{2} & \text{if } 1 < x \leq 4 \end{cases}$

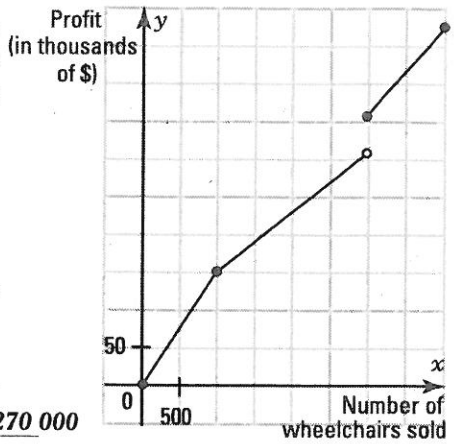


c) $f_3(x) = \begin{cases} -2 & \text{if } x \leq -1 \\ -x^2 & \text{if } 0 \leq x \leq 2 \\ 2x - 4 & \text{if } x > 2 \end{cases}$



2. The company Kandev sells wheelchairs for retirement homes. The function f which associates the number x of wheelchairs sold with the net annual profit y (in thousands of dollars) has the rule:

$$y = \begin{cases} 0.15x & 0 \leq x \leq 1000 \\ 0.08x + 70 & 1000 < x < 3000 \\ 0.12x & 3000 \leq x \leq 4000 \end{cases}$$



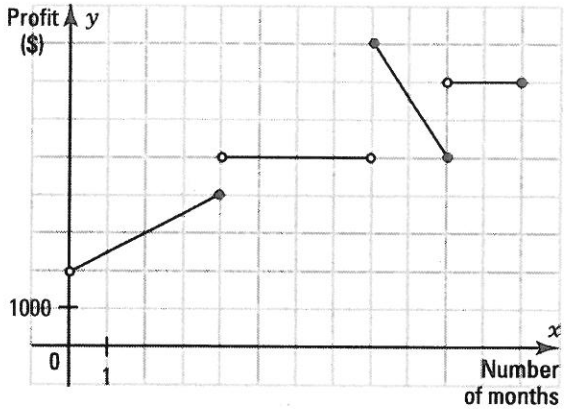
a) If the maximum number of wheelchairs sold per year is 4000, draw the graph of this function.

b) Find dom f. ***[0, 4000]***

c) How much profit is made by selling 2500 wheelchairs? ***$270 000***

d) Over which interval is the rate of change the greatest? ***[0, 1000]***

3. The piecewise function defined on the right associates the number x of months since the beginning of the year with the profit $f(x)$ of a company during that year.



a) What is the company's profit after

1. 2 months? ***$3000*** 2. 4 months? ***$4000***
3. 6 months? ***$5000*** 4. 11 months? ***$7000***

b) Determine the number of months since the beginning of the year if the company's profit is

1. $3000. ***2 months***
2. $6500. ***9 months***

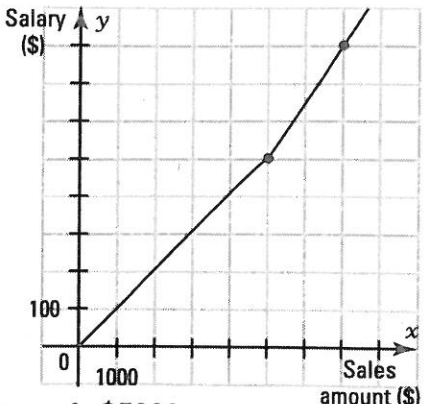
c) Determine the rule of function f.

$$f(x) = \begin{cases} 500x + 2000 & \text{if } 0 < x \leq 4 \\ 5000 & \text{if } 4 < x < 8 \\ -1500x + 20\,000 & \text{if } 8 \leq x \leq 10 \\ 7000 & \text{if } 10 < x \leq 12 \end{cases}$$

d) Over what interval is function f

1. increasing? ***]0, 4]***
2. decreasing? ***[8, 10]***
3. constant? ***]4, 8[or]10, 12]***

4. The function f represented on the right associates the amount x of sales made in a week by a salesman at a company with the salary y he received.



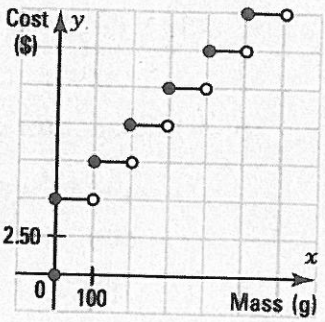
a) Find the rule of f.

$$f(x) = \begin{cases} 0{,}10x & 0 \leq x < 5000 \\ 0{,}15x - 250 & x \geq 5000 \end{cases}$$

b) Explain how the company calculates the salary of a salesman.

A salesman receives 10% of the sales amount when it is less than $5000. He receives 15% of the sales amount less than $250 of expenses when it is equal to or exceeds $5000.

5. The cost (in \$) of sending a parcel depends on its mass (in g). The cost is \$5 for a mass less than 100 g and \$2.50 for each additional 100 g.

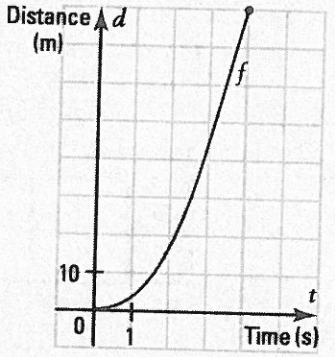


a) Draw the graph of the function which associates the mass x of the parcel with the cost y of sending it.

b) What is the cost of sending a 325 g parcel? ***\$12.50***

c) In what interval lies the mass of a parcel if it costs \$15 to send it? ***[400, 500[***

6. From the top of 80 m tall building, an object is thrown vertically downward. The function f which associates the time t (in s) elapsed since the start with the distance d traveled (in m) has the rule: $d = 5t^2$.



a) Represent function f in the Cartesian plane on the right.

b) At what time t does the object hit the ground? ***4 s***

c) Determine in this situation

1. dom f. ***[0, 4]*** 2. ran f. ***[0, 80]***

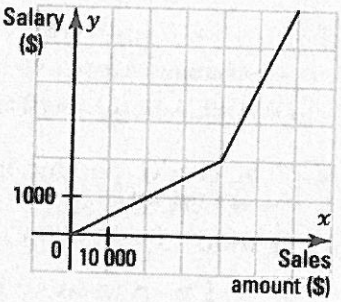
7. A herd presently contains 7 elephants. This herd doubles every 6 years. After how many years will the herd contain 112 elephants? ***After 24 years***

8. A capital of \$1000 is invested during 5 years at an interest rate of 10% compounded annually. Determine the accumulated capital. ***$y = 1000(1.10)^5 = \$1610.51$***

9. A ball bounces to a height equal to $\frac{3}{5}$ of the height reached with the previous rebound. The ball is dropped from a 25 m tall building. What height does the ball reach after the sixth rebound? ***1.17 m***

10. The monthly salary y of an employee depends on the amount of sales made during the month. The function f which gives the employee's salary has rule:

$$f(x) = \begin{cases} 0.05x & 0 \le x < 40\,000 \\ 0.2x - 6000 & x \ge 40\,000 \end{cases}$$



a) Represent the function in the Cartesian plane on the right.

b) What is the salary of an employee who makes \$30 000 in sales in a month? ***\$1500***

c) What is the amount of sales made by an employee who receives a salary of \$4700? ***\$53 500***