2.6 Exponential function

ACTIVITY 1 Rational exponents

a) 1. Calculate the following powers.

1) 5^2 **25** 2) $(-2)^4$ **16** 3) $(-4)^3$ **-64** 4) 2^3 **8**

2. Let a be a real number and n an integer greater than 1.
Under which conditions is the power a^n negative? **$a < 0$ and n odd**

b) 1. Calculate the following powers.

1) 2^{-3} **$\frac{1}{8}$** 2) $(-2)^{-2}$ **$\frac{1}{4}$** 3) $\left(\frac{3}{2}\right)^{-3}$ **$\frac{8}{27}$** 4) $\left(-\frac{3}{2}\right)^{-2}$ **$\frac{4}{9}$**

2. Let a be a non zero real number and n an integer, complete: $a^{-n} =$ **$\frac{1}{a^n}$**

c) 1. Calculate, whenever possible, the following powers.

1) $16^{\frac{1}{2}}$ **4** 2) $16^{\frac{1}{4}}$ **2** 3) $(-8)^{\frac{1}{3}}$ **-2** 4) $(-16)^{\frac{1}{2}}$ ***impossible***

2. Let a be a real number and n an integer greater than 1.
Under which conditions does $a^{\frac{1}{n}}$ not exist in $\mathbb{R}$? **$a < 0$ and n even**

RATIONAL NUMBER EXPONENTS

If a is a real number and n an integer greater than 1, we have:

$$\boxed{a^{\frac{1}{n}} = \sqrt[n]{a}}$$

Ex.: $16^{\frac{1}{2}} = \sqrt{16} = 4$; $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$; $(-27)^{\frac{1}{3}} = \sqrt[3]{-27} = -3$

Note that $a^{\frac{1}{n}}$ does not exist in $\mathbb{R}$ when $a < 0$ and n is even.

ACTIVITY 2 Exponential growth

The number of cells in a controlled environment doubles every day. Initially ($x = 0$), we observe one million cells.

x	−2	−1	0	1	2	3
y	***0.25***	***0.5***	1	***2***	***4***	***8***

a) Complete the table of values of the function which associates the number of days x since the beginning with the number y, in millions, of cells observed.

b) What is the rule of the function? **$y = 2^x$**

c) How many cells do we observe

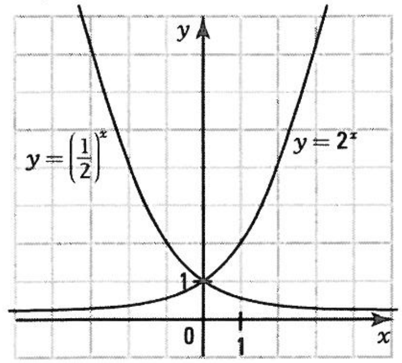
1. 5 days after the beginning? ***32 million*** 2. 3 days before the beginning? ***125 000***

d) The number of cells increases very rapidly. We say that the growth is exponential. At what time do we observe

1. 128 million cells? ***7 days after the beginning*** 2. 62 500 cells? ***4 days before the beginning***

ACTIVITY 3 Basic exponential function $y = c^x$

Consider the function $y = 2^x$ and $y = \left(\frac{1}{2}\right)^x$ where $x \in \mathbb{R}$.



a) For each function complete the table of values.

1.

x	−2	−1	0	1	2	3
$y = 2^x$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

2.

x	−2	−1	0	1	2	3
$y = \left(\frac{1}{2}\right)^x$	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

b) Represent each function in the Cartesian plane.

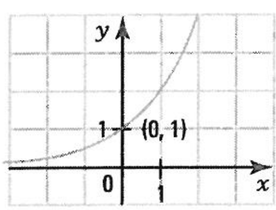
We observe that each of the curves representing these function gets closer to the x-axis without ever touching it.

We say that the x-axis is an ***asymptote*** *for the curve or that the curve is asymptotic to the x-axis.*

EXPONENTIAL FUNCTION $y = c^x$

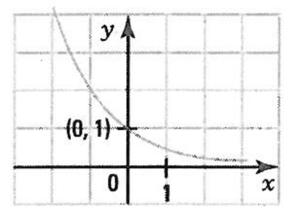
- The function $f(x) = c^x$, where c is a positive real number different from 1, is called exponential function in base c.
  This function describes exponentially increasing or decreasing situations depending on whether the base is greater or less than 1.

$c > 1$



Increasing function

$0 < c < 1$



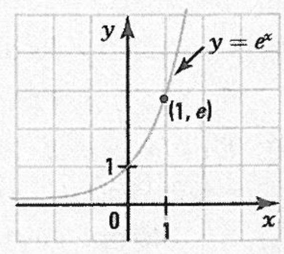
Decreasing function

- For any base, we have:
  - dom $f = \mathbb{R}$ and ran $f = \mathbb{R}_+^*$.
  - The function does not have any zeros.
  - The x-axis is an asymptote of the curve.
  - The initial value is equal to 1 ($c^0 = 1$).
  - The function is positive over $\mathbb{R}$.
- When the independent variable x increases by 1 unit, the dependent variable y is multiplied by the multiplicative factor c (base of the exponential function) called periodic multiplicative factor.

	$\xrightarrow{+1}$	$\xrightarrow{+1}$	$\xrightarrow{+1}$	
x	0	1	2	3	…
y	1	c	c^2	c^3	…
	$\xrightarrow{\times c}$	$\xrightarrow{\times c}$	$\xrightarrow{\times c}$	

- The most common bases for the exponential function are the numbers 10 and e.
  e is an irrational number that occurs in phenomena related to physics, biology, calculation of probabilities, …

$$\boxed{e = 2.71828\ldots}$$



1. For each of the following exponential functions $y = c^x$,

1. determine the base.
2. indicate if the function is increasing or decreasing.

a) $f(x) = \left(\frac{4}{3}\right)^x$ ***$\frac{4}{3}$; increasing***

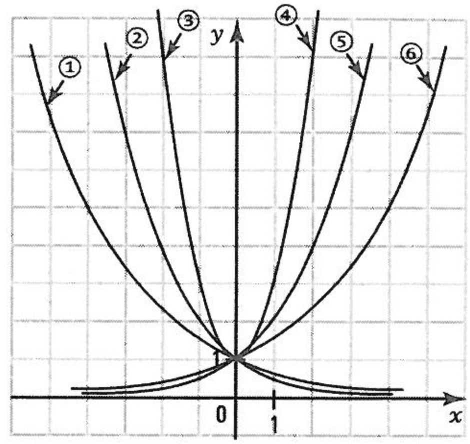
b) $f(x) = \left(\frac{4}{5}\right)^x$ ***$\frac{4}{5}$; decreasing***

c) $f(x) = 2^{-x}$ ***$\frac{1}{2}$; decreasing***

d) $f(x) = \left(\frac{2}{3}\right)^{-x}$ ***$\frac{3}{2}$; increasing***

2. On the right are represented the exponential functions with equations:

$$y = 2^x, y = \left(\frac{3}{2}\right)^x, y = 3^x, y = \left(\frac{1}{2}\right)^x, y = \left(\frac{2}{3}\right)^x, y = \left(\frac{1}{3}\right)^x.$$



a) Match each curve with its equation.

1. $y = \left(\frac{2}{3}\right)^x$ 2. $y = \left(\frac{1}{2}\right)^x$ 3. $y = \left(\frac{1}{3}\right)^x$

4. $y = 3^x$ 5. $y = 2^x$ 6. $y = \left(\frac{3}{2}\right)^x$

b) Of the three increasing exponential functions, which one increases the fastest? Justify your answer.

$y = 3^x$. It is the one with the largest base.

c) Of the three decreasing exponential functions, which one decreases the fastest? Justify your answer.

$y = \left(\frac{1}{3}\right)^x$. It is the one with the smallest base.

ACTIVITY 4 Exponential function $y = ac^x$ – Role of parameter a

A bacterial culture contains 500 bacteria initially. The number of bacteria doubles every hour. Consider the function f, which associates, to the elapsed time t in hours, the number of bacteria present in the culture.

a) Complete the following table of values on the right.

t (h)	y
0	500
1	***1 000***
2	***2 000***
3	***4 000***

b) Determine the parameter a and the base c of the rule of the function $y = ac^t$.

$y = ac^t$ $y = 500c^t$ $y = 500(2)^t$

$500 = ac^0$ $1\ 000 = 500c^1$

$a = 500$ $c = 2$

c) Interpret the role of parameter a. ***$a = 500$ represents the initial value of the function. The culture contains 500 bacteria initially.***

ACTIVITY 5 Exponential function $y = ac^x$ – Interpretation of parameter a

The value $c(t)$ after t years of a capital a invested at an interest rate of i compounded annually is calculated according to the rule: $\boxed{c(t) = a(1 + i)^t}$

A $1000 capital is invested at the bank at an interest rate of 8% compounded annually.

t	$c(t)$
0	1000
1	1080
2	***1166.40***
3	***1259.71***
⋮	

a) Complete the table of values which associates the number t of years with the value $c(t)$.

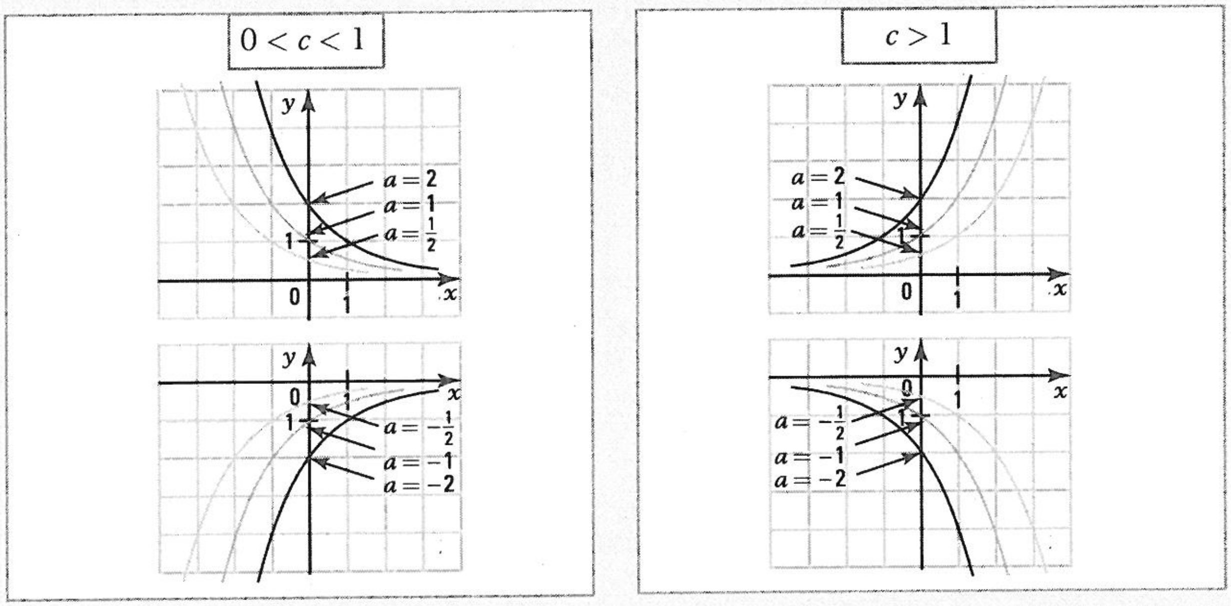
b) What is the rule of this exponential function? ***$c(t) = 1000(1.08)^t$***

c) What is the base of the exponential function? ***1.08***

d) Determine and interpret the role of parameter a.
a = 1000. a represents the initial capital (t = 0), $c(0) = a$ = $1000.

EXPONENTIAL FUNCTION $y = ac^x$ – ROLE OF PARAMETER a

- The exponential curve undergoes a vertical stretch when the absolute value of parameter a increases.
- Parameter a corresponds to the initial value of the function.



3. Julie deposits $1000 at the bank at an interest rate of 8% compounded annually. The capital $C(t)$, accumulated after t years is given by $C(t) = 1000(1.08)^t$. What is the accumulated capital after

a) 3 years? ***$1259.71*** b) 5 years? ***$1469.33***

4. A radioactive substance decays over time. Its mass m (in grams) is expressed as a function of time t (in years) by the equation $m = 10(0.8)^t$. What is the mass of this substance

a) today ($t = 0$)? ***10 g*** b) in 2 years? ***6.4 g*** c) 1 year ago? ***12.5 g***

5. The value $v(t)$ of a car which depreciates by 20% per year is given by $v(t) = v_0(0.80)^t$ where v_0 represents the purchase cost of the new car and t the number of years since it was bought.

a) What is the value of a car 3 years after it was bought if, new, the cost is \$30 000?

\$15 360

b) What was the purchase cost of a car that is worth \$22 400 two years after it was bought?

\$35 000

ACTIVITY 6 Exponential function $y = c^{bx}$ – Role of parameter b ($b < 1$)

A bacterial culture contains 100 bacteria initially. The number of bacteria doubles every 15 minutes. Consider the function *f*, which associates, to the elapsed time *t* in hours, the number of bacteria present in the culture.

a) Complete the following table of values on the right.

b) Determine the parameter a and the base c of the rule of the function $y = ac^t$.

t (h)	y
0	100
0.25	***200***
0.5	***400***
0.75	***800***
1	***1600***

$y = ac^t$ $\quad y = 100c^t$ $\quad y = 100(16)^t$

$100 = ac^0$ $\quad 200 = 100c^{\frac{1}{4}}$

$a = 100$ $\quad c^{\frac{1}{4}} = 2$

$c = 16$

c) The number of bacteria doubles every 15 minutes. Write the rule obtained in b) in the form $y = a(2)^{bt}$.

$y = 100(16)^t$ $\quad y = 100(2)^{4t}$

d) Interpret the role of parameter b.

The number of bacteria doubles every 15 minutes. 15 min = $\frac{1}{4}$h. b = 4. The parameter b corresponds to the number of periods per unit time (hour).

ACTIVITÉ 7 Exponential function $y = ac^{bx}$ – Role of parameter b ($b < 1$)

A forest contains approximately 2000 trees. Following a reforestation program, an increase of the number of trees by 15% every 2 years is predicted.

a) Complete the following table of values on the right.

b) Determine the parameter a and the base c of the rule of the function $y = ac^t$.

t (years)	y
0	2000
2	***2300***
4	***2645***

$y = ac^t$ $\quad y = 2\,000c^t$ $\quad y = 2\,000(1,15)^{\frac{1}{2}t}$

$2\,000 = ac^0$ $\quad 2\,300 = 2\,000c^2$

$a = 2\,000$ $\quad c^2 = 1,15$

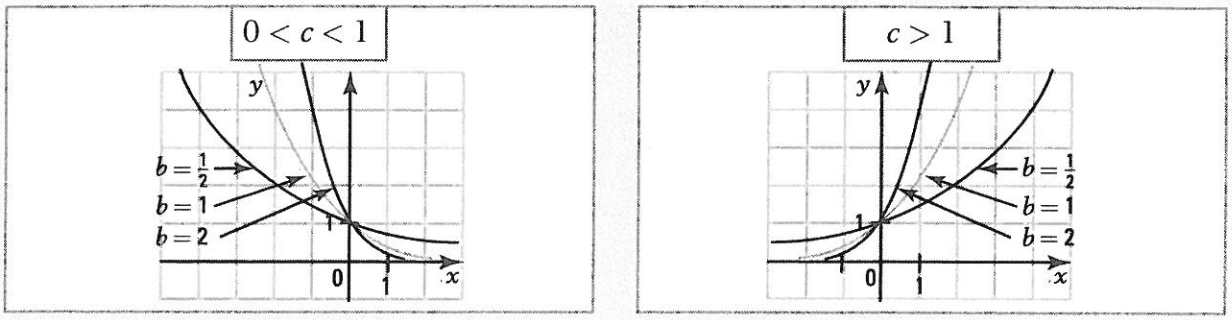
$c = (1,15)^{\frac{1}{2}}$

c) Interpret the role of parameter b.

$b = \frac{1}{2}$. The number of trees increases by 15% every 2 years. The parameter b is the number of periods per unit time (years).

EXPONENTIAL FUNCTION $y = c^{bx}$ – ROLE OF PARAMETER b

- When parameter b increases, the exponential curve undergoes a horizontal reduction, for any base.
- If x represents the elapsed time, parameter b corresponds to the number of periods per time unit.



6. The number of bacteria in an environment doubles every 15 minutes on average. Initially, the environment contains 1 thousand cells. Consider the function which associates the number t of hours since the beginning with the number $N(t)$, in thousands, of cells in the environment.

a) Complete the table of values on the right.

t	0	0.25	0.50	0.75	1	2
$N(t)$	1	2	4	8	16	256

b) What is the rule of the function? $y = 2^{4t}$

c) What is the number of cells after 3 hours? 4096 thousand

d) After how many hours will the environment contain 1024 thousand cells? 2 h 30 min

ACTIVITY 8 Finding the rule $y = ac^x$

For a reforestation project, a company decides to double the number of trees each year. At the beginning, the forest contained 25 trees.

a) Find the rule of the function which associates the number x of years since the beginning with the number y of trees in the forest. $y = 25(2)^x$

b) How many trees will there be after 7 years? 3200 trees

ACTIVITY 9 Finding the rule $y = ac^{bx}$

A controlled environment initially contains 3 bacteria. The number of bacteria doubles every 15 minutes. Find the rule of the exponential function which associates the time, in hours, since the beginning with the number y of bacteria in the environment. The rule is of the form: $y = ac^{bx}$.

a) Complete the table of values on the right.

x	y
0	3
$\frac{1}{4}$	6
$\frac{1}{2}$	12
$\frac{3}{4}$	24
1	48
2	758

b) The initial value of the function represented by a is the value at the beginning ($x = 0$). What is the initial value a? $a = 3$

c) The number of bacteria doubles every 15 minutes. What is the multiplicative factor c? $c = 2$

d) The time x since the beginning being expressed in hours, parameter b represents the number of 15-minute periods per hour.

1. What is the value of parameter b? $b = 4$
2. Therefore, what is the rule of the function in this situation? $y = 3(2)^{4x}$

e) What would be the rule if the number of bacteria doubled

1. every twenty minutes? $y = 3(2)^{3x}$
2. every two hours? $y = 3(2)^{\frac{x}{2}}$

RULE OF AN EXPONENTIAL FUNCTION $y = ac^{bx}$

- The time unit being well defined, the independent variable x represents the time elapsed since the beginning ($x = 0$). The dependent variable y (number of bacteria, accumulated capital, ...) takes on values which are periodically multiplied by a constant factor.
  a represents the initial value of the dependent variable.
  b represents the number of periods per unit of time.
  c represents the periodic multiplicative factor, which multiplies the variable y at each period.
  Ex.: An environment initially contains 10 bacteria. The time unit is the hour.
  - When the number of bacteria triples ($c = 3$) every 15 minutes, each hour contains 4 periods ($b = 4$). The rule is $y = 10(3)^{4x}$.
  - When the number of bacteria doubles ($c = 2$) every 30 minutes, each hour contains 2 periods ($b = 2$). The rule is $y = 10(2)^{2x}$.
  - When the number of bacteria quadruples ($c = 4$) every 2 hours, each hour contains $\frac{1}{2}$ period $\left(b = \frac{1}{2}\right)$. The rule is $y = 10(4)^{\frac{1}{2}x}$.

7. Each of the following situations is described by an exponential function with rule $y = ac^{bx}$. After establishing the time unit,

1. define the variables x and y. 2. determine the parameters a, b and c.
3. find the rule of the function.

a) In an environment containing 1000 bacteria initially, the number of bacteria triples every 10 minutes.

1. x: number of hours since the beginning, y: number of bacteria in the environment.

2. $a = 1000$, $b = 6$, $c = 3$ 3. $y = 1000(3)^{6x}$

b) In a habitat initially containing 100 insects, the number of insects doubles every 3 days.

1. x: number of days since the beginning, y: number of insects in the habitat.

2. $a = 100$, $b = \frac{1}{3}$, $c = 2$ 3. $y = 100(2)^{\frac{1}{3}x}$

c) A car bought for \$30 000 loses 20% of its value each year.

1. x: number of years since the car was bought, y: value of the car

2. $a = 30\ 000$, $b = 1$, $c = 0.80$ 3. $y = 30\ 000(0,80)^{x}$

d) An initial population of 1000 deer increases by 15% every year.

1. x: number of years since the beginning, y: deer population

2. $a = 1000$, $b = 1$, $c = 1.15$ 3. $y = 1000(1.15)^{x}$

e) A 50 g radioactive mass loses half its mass every 6 hours starting at noon.

1. x: number of hours since noon, y: mass remaining

2. $a = 50$; $b = \frac{1}{6}$, $c = \frac{1}{2}$ 3. $y = 50\left(\frac{1}{2}\right)^{\frac{x}{6}}$

8. A grocery basket costs \$180 today. If an annual inflation rate of 2.4% is predicted, how much will this basket cost in 8 years?

$y = 180(1.024)^t$; $y = 180(1.024)^8 = \$217.61$

9. Mr. Jasmin invests \$4800 in a bank account offering an interest rate of 2.5% compounded annually. What will be the value of the investment after 5 years?

$y = 4\,800(1.025)^t$; $y = 4\,800(1.025)^5 = \$5\,430.76$

10. The value of a car depreciates by 25% per year. If a car was purchased for \$28 500, what will be its value after 4 years?

$y = 28\,500(0.75)^t$; $y = 28\,500(0.75)^4 = \$9\,017.58$

11. A bacterial culture contains 2000 bacteria. Determine the number of bacteria in the culture after 6 hours if the number of bacteria

a) increases by 35% per hour

$y = 2\,000(1.35)^t$; $y = 2\,000(1.35)^6 = 12\,107$ bacteria

b) decreases by 20% every 30 minutes

$y = 2\,000(0.8)^{2t}$; $y = 2\,000(0.8)^{2(6)} = 137$ bacteria

c) doubles every 3 hours

$y = 2\,000(2)^{\frac{1}{3}t}$; $y = 2\,000(2)^{\frac{1}{3}(6)} = 8\,000$ bacteria

d) increases by 4% every 10 minutes.

$y = 2\,000(1.04)^{6t}$; $y = 2\,000(1.04)^{6(6)} = 8\,208$ bacteria

12. The population of a city increases by 10% every 2 years. What is the population of this city if it is expected that after 4 years it will be of 24 200 inhabitants?

$y = a(1.1)^{\frac{1}{2}t}$; $24\,200 = a(1.1)^{\frac{1}{2}(4)}$; $a = 20\,000$ inhabitants

13. The value of a car depreciates by 38% every 3 years. How much was a car purchased if after 2 years its value is \$22 000?

$y = a(0.62)^{\frac{1}{3}t}$; $22\,000 = a(0.62)^{\frac{1}{3}(2)}$; $a = \$30\,257.16$

14. An amount of \$3000 is invested in a bank account at an interest rate compounded annually. The accumulated value of this investment after 2 years is \$3121.20. What will be the accumulated value of the investment 10 years from the beginning of the initial investment?

$y = 3\,000c^t$; $3\,121.20 = 3\,000c^2$; $c^2 = 1.0404$; $c = 1.02$. The annual interest rate is 2%.

$y = 3\,000(1.02)10 = \$3\,656.98$. After 10 years, the investment value is \$3656.98.

ACTIVITY 10 Exponential equation – Model $c^u = c^v$

a) Justify the steps when solving the equation $3(2)^{4x} = 768$.

$3(2)^{4x} = 768$

$\Leftrightarrow \quad 2^{4x} = 256$ ***We divide each member by 3.***

$\Leftrightarrow \quad 2^{4x} = 2^8$ ***We write the right-hand side member as a power of 2.***

$\Leftrightarrow \quad 4x = 8$ ***We deduce the equality of the exponents.***

$\Leftrightarrow \quad x = 2$ ***We deduce the unknown x.***

b) The number $P(t)$ of bacteria in an environment is given by $P(t) = 100(2)^{3t}$ where t represents the number of days since the beginning. How many days after the beginning do we observe 6400 cells?

$100(2)^{3t} = 6400$

$2^{3t} = 64$

$2^{3t} = 2^6$

$t = 2$. We observe 6400 cells 2 days after the beginning.

EXPONENTIAL EQUATION – MODEL $c^u = c^v$

When the members of an exponential equation can be written as powers of the same base, we use the logical equivalence:

$$c^u = c^v \Leftrightarrow u = v$$

	$5(2)^{3x} = 320$
1. We isolate the power containing the unknown in one member.	$2^{3x} = 64$
2. We write an equality between two powers of the same base.	$2^{3x} = 2^6$
3. We equate the exponents.	$3x = 6$
4. We determine the unknown.	$x = 2$
5. We establish the solution set S.	Therefore, $S = \{2\}$.

15. Solve the following exponential equations.

a) $3^x = 243$ — ***x = 5***

b) $2^x = \frac{1}{8}$ — ***x = −3***

c) $2(5)^x = 250$ — ***x = 3***

d) $\left(\frac{2}{3}\right)^x = \frac{27}{8}$ — ***x = −3***

e) $4^3 = 8^x$ — ***x = 2***

f) $\left(\frac{1}{2}\right)^x = 32$ — ***x = −5***

g) $5^{2x} - 1 = 0$ — ***x = 0***

h) $2(5)^x - 48 = 2$ — ***x = 2***

i) $9^x - 27 = 0$ — $x = \frac{3}{2}$

16. The growth of a herd of bison follows the rule $P = 400(2)^{\frac{t}{10}}$ where P is the population after t years. After how many years will the population be equal to four times the initial population?

$1600 = 400(2)^{\frac{t}{10}} \Leftrightarrow 2^{\frac{t}{10}} = 4 \Leftrightarrow 2^{\frac{t}{10}} = 2^2 \Leftrightarrow \frac{t}{10} = 2 \Leftrightarrow t = 20$. ***After 20 years.***

17. A population of mosquitoes doubles every seven days. If there are initially 5 mosquitoes, after how many days will the population contain 80 mosquitoes?

$5(2)^{\frac{t}{7}} = 80 \Leftrightarrow 2^{\frac{t}{7}} = 16 \Leftrightarrow t = 28$. ***After 28 days.***

18. A radioactive mass decays according to the rule $m(t) = 100\left(\frac{1}{2}\right)^{\frac{t}{4}}$ where $m(t)$ is the mass remaining after t hours.

a) Determine after how many hours the remaining mass will be equal to 25 g. ***8 hours***

b) The time necessary for the mass of a radioactive substance to be reduce by half because of decay is called the **half-life** of the substance. What is the half-life of this substance? ***4 hours***

19. The number of bacteria in culture A containing 50 bacteria initially doubles every 3 hours and that in culture B containing 20 bacteria initially increases by 20% every 30 minutes. How many bacteria will be present in culture B at the moment when culture A will contain 800 bacteria?

Culture A : $y = 50(2)^{\frac{1}{3}t}$; $800 = 50(2)^{\frac{1}{3}t}$; $2^{\frac{1}{3}t} = 16$; $2^{\frac{1}{3}t} = 2^4$; $t = 12$

Culture B : $y = 20(1{,}2)^{2t} = 20(1{,}2)^{2(12)} = 1590$ bacteria. ***Culture B will contain 1590 bacteria when culture A will contain 800 bacteria.***

Evaluation 2

1. Find the rule of the following functions.

a) f is a constant function such that $f(1) = 2$. ***$f(x) = 2$***

b) f is linear function of direct variation such that $f(1) = 2$. ***$f(x) = 2x$***

c) f is a linear function such that $f(1) = 2$ and $f(3) = 5$. ***$f(x) = \frac{3}{2}x + \frac{1}{2}$***

d) f is a function for which the product of the variables x and y is equal to 12. ***$f(x) = \frac{12}{x}$***

e) f is a quadratic function whose graph has vertex V(0, 0) and such that $f(2) = 6$.

$f(x) = \frac{3}{2}x^2$

f) f is an exponential function whose graph passes through the points (0, 5) and (2, 20)

$f(x) = 5(2)^x$

2. For each of the following tables of values, find the rule of the function associated with it.

a)

x	0	1	2	3
y	16	8	4	2

$y = 16\left(\frac{1}{2}\right)^x$

b)

x	0	1	2	3
y	−2	−2	−2	−2

$y = -2$

c)

x	1	2	3	4
y	12	6	4	3

$y = \frac{12}{x}$

d)

x	0	1	2	3
y	0	0.25	1	2.25

$y = \frac{1}{4}x^2$

e)

x	0	1	2	3
y	8	12	18	27

$y = 8\left(\frac{3}{2}\right)^x$

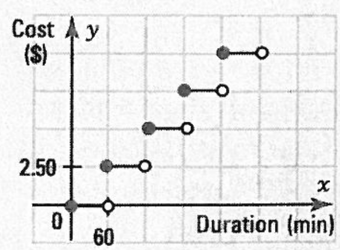
f)

x	2	4	6	8
y	1	4	7	10

$y = \frac{3}{2}x - 2$

3. A the beginning of a car trip, the gas tank contains 66 litres. After traveling 50 km, the tank contains 60 litres. What is the rule of the linear function which associates the distance x traveled, in kilometres, with the quantity y, in litres, of gas left in the tank? ***$y = 66 - 0.12x$***

4. The cost of parking in a garage is represented in the Cartesian plane on the right. Consider the function which associates the parking duration, in minutes, with the cost y in dollars.



a) Describe, using words, how the cost is calculated.

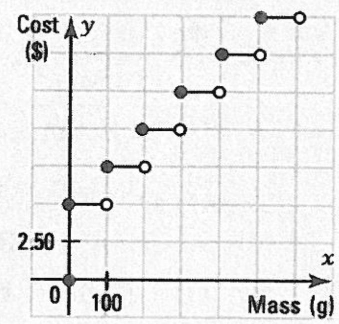
Parking is free for a duration less than 60 minutes.

There are charges of $2.50 for each additional complete 60-minute period.

b) What is the cost for 2 h 15 min? ***$5***

c) What is the possible duration if we pay $12.50? ***$300 \leqslant x < 360$***

5. The cost (in \$) of sending a parcel depends on its mass (in g). The cost is \$5 for a mass less than 100 g and \$2.50 for each additional 100 g.

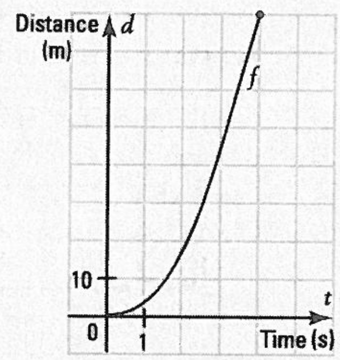


a) Draw the graph of the function which associates the mass x of the parcel with the cost y of sending it.

b) What is the cost of sending a 325 g parcel? ***\$12.50***

c) In what interval lies the mass of a parcel if it costs \$15 to send it? ***[400, 500[***

6. From the top of 80 m tall building, an object is thrown vertically downward. The function f which associates the time t (in s) elapsed since the start with the distance d traveled (in m) has the rule: $d = 5t^2$.



a) Represent function f in the Cartesian plane on the right.

b) At what time t does the object hit the ground? ***4 s***

c) Determine in this situation

1. dom f. ***[0, 4]*** 2. ran f. ***[0, 80]***

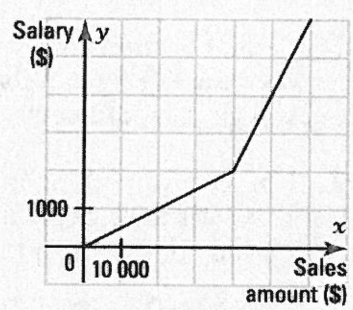
7. A herd presently contains 7 elephants. This herd doubles every 6 years. After how many years will the herd contain 112 elephants? ***After 24 years***

8. A capital of \$1000 is invested during 5 years at an interest rate of 10% compounded annually. Determine the accumulated capital. ***$y = 1000(1.10)^5$ = \$1610.51***

9. A ball bounces to a height equal to $\frac{3}{5}$ of the height reached with the previous rebound. The ball is dropped from a 25 m tall building. What height does the ball reach after the sixth rebound? ***1.17 m***

10. The monthly salary y of an employee depends on the amount of sales made during the month. The function f which gives the employee's salary has rule:

$$f(x) = \begin{cases} 0.05x & 0 \leq x < 40\,000 \\ 0.2x - 6000 & x \geq 40\,000 \end{cases}$$



a) Represent the function in the Cartesian plane on the right.

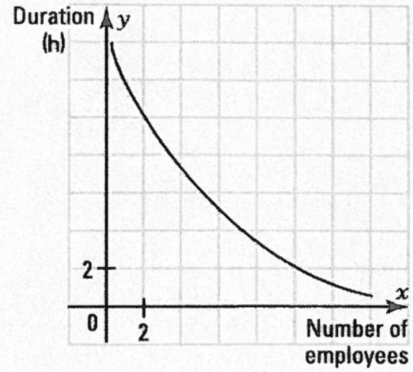
b) What is the salary of an employee who makes \$30 000 in sales in a month? ***\$1500***

c) What is the amount of sales made by an employee who receives a salary of \$4700?
\$53 500

11. A company wishes to do renovation work in a building in order to reduce energy costs. The renovations require a total of 20 hours.

a) Draw the graph of the function f which associates the number x of employees with the duration y of the renovations.

b) What is the rule of this function? $y = \frac{20}{x}$



12. Aurelia and Raphael purchase the same day a car at a dealership. The value of Aurelia's car depreciates by 20% per year and the value of Raphael's car depreciates by 14% per year. Aurelia purchased her car for \$24 000 and Raphael purchased his for \$28 500.

What will be the value of Raphael's car at the time that Aurelia's car is worth \$15 360?

Aurelia: $y = 24\,000(0.8)^t$; $15\,360 = 24\,000(0.8)^t$; $0.8^t = 0.64$; $t = 2$

Raphael: $y = 28\,500(0.86)^2 = \$21\,078.60$

13. An epidemic reaches a village in Africa. At the beginning of the epidemic, 400 infected people are counted. Every month, this number increases by 12% compared to the previous month. Six months after the beginning of the epidemic, it ceases to spread and the infected population decreases by 20% every 4 months.

What is the number of people infected 16 months after the beginning of the epidemic?

After 6 months: $y = 400(1.12)^t$; $y = 400(1.12)^6 = 790$ ***infected people***

After 16 months: $y = 790(0.8)^{\frac{1}{4}t} = 790(0.8)^{\frac{1}{4}(12)} = 404$ ***infected people***

14. Sylvia is a seller in a store of household appliances. She receives a commission of \$120 for every \$1000 of sales made.
In January, she received a \$360 commission. If, in February, she sells twice as much as in January, will she receive double the commission? Justify your answer.

No

January: amount of sales ∈ [3 000, 4 000[

For exemple: \$3800 in sales

Commission: \$360

February: amount of sales \$7600 (2 x \$3800)

Commission: \$840