• **Cartesian graph**
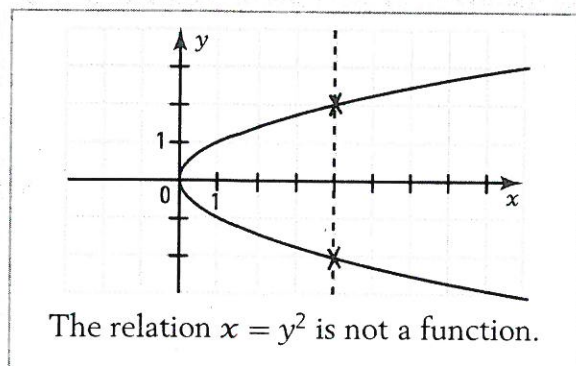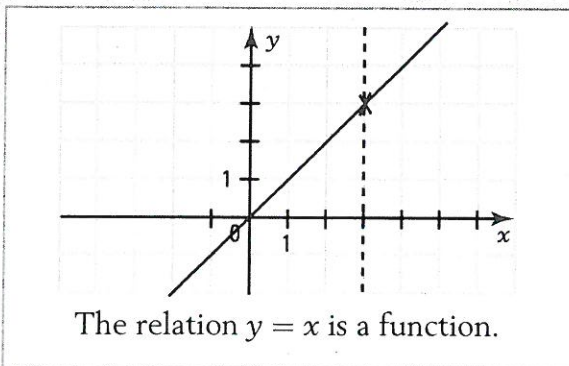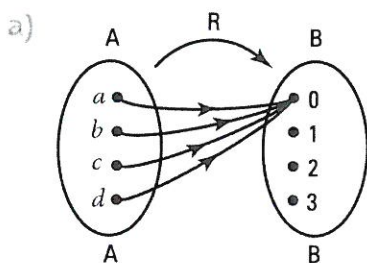
Given the Cartesian graph of a relation, this relation is a function if any vertical line intersects the graph of this relation in at most one point.
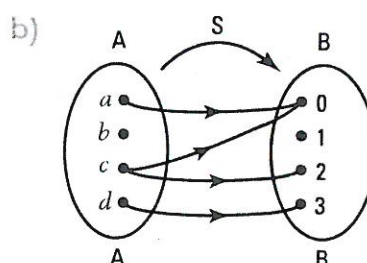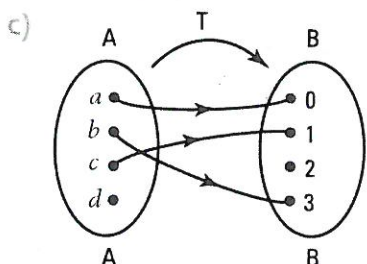
Ex.:

The relation $y = x$ is a function.

The relation $x = y^2$ is not a function.

1. In each of the following cases, indicate if the relation is a function. If not, explain why.

a)

Yes

b)

No, since c is in relation with 2 elements: 0 and 2.

c)

Yes

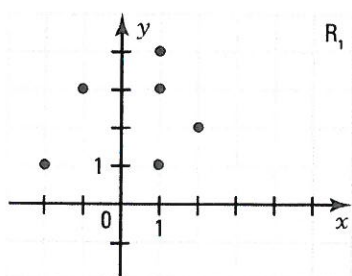d)
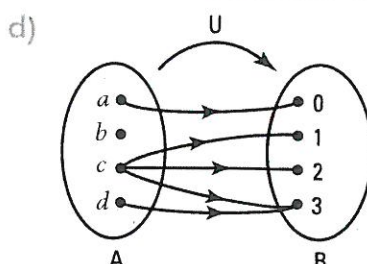
No, since c is in relation with 3 elements: 1, 2 and 3.

in new workbook →

2. Determine if the following relations are functions.

a)

No

b)

Yes

c)

Yes

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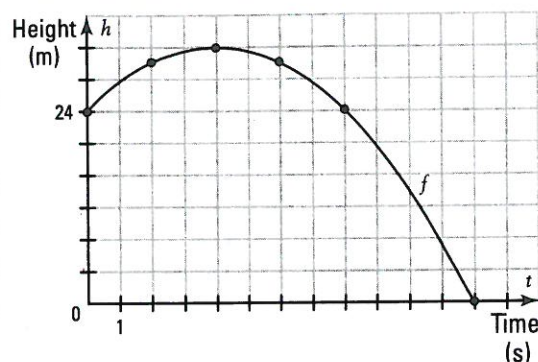**1.1** Properties of functions   **5**
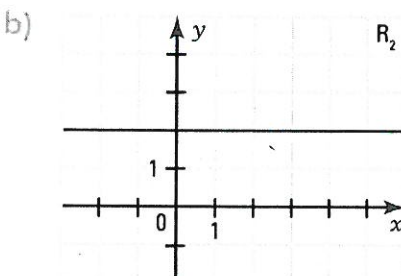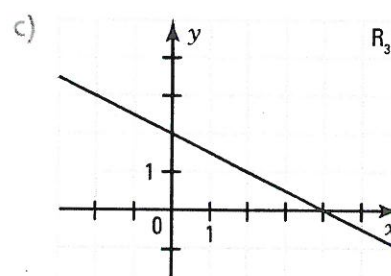
A projectile is launched from a height of 24 m above the ground. The graph on the right represents the height h (in metres) as a function of time t (in seconds) since it was launched.

a) 1. After how many seconds does the projectile hit the ground? _____ 12 s _____

2. Over what interval does the variable t take its values in this situation? _____ [0, 12] _____

This interval is called the **domain** of function f.

b) 1. What is the maximum height reached by the projectile? _____ 32 m _____
2. What is the minimum height reached by the projectile? _____ 0 m _____
3. Over what interval does the variable h take its values in this situation? _____ [0, 32] _____

This interval is called the **range** of function f.

DOMAIN AND RANGE

- The **domain** of a function is the set of values taken by the independent variable.

- The **image** or **range** of a function is the set of values taken by the dependent variable.

Ex.: A tank contains 200 litres of water. During 5 hours, using a pump, water is added to the tank at a rate of 50 litres per hour. The function f associates, to the time t elapsed since the beginning, the quantity q of the water present in the tank.

dom f = [0, 5]. The independent variable t takes its values in the interval [0, 5].

ima f = [200, 450]. The dependent variable q takes its values in the interval [200, 450].

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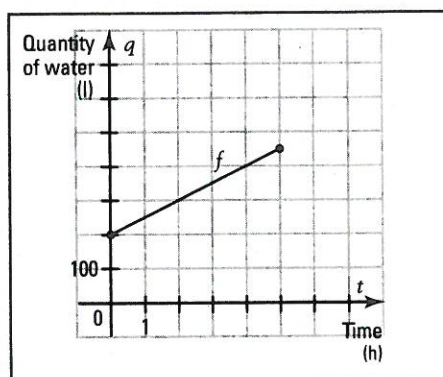
3. Emilien takes a taxi to go to work, which is 8 kilometers away from home. The driver charges a fixed amount of $4.50 plus an additional amount of $0.75 per kilometer travelled. The graph on the right represents the function f, which associates, to the distance travelled in kilometers, the cost in $.
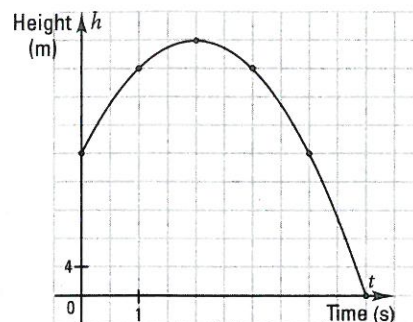
a) What is the domain of this function? _____ [0, 8] _____
b) What is the range of this function? _____ [4.50, 10.50] _____
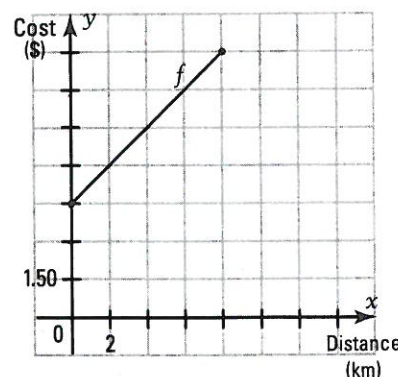
4. From the top of a hill 20 m high, a projectile is launched upwards. The graph on the right associates, to the time t in seconds, the height h of the projectile in meters.

a) Determine and interpret the domain of this function.

_____

b) Determine and interpret the range of this function.

_____

_____

5. The stock of Hydrobec was just released on the market. A broker observes the value of this stock for 6 months. The graph on the right associates, to the number of months since the stock was released, the value of the stock.
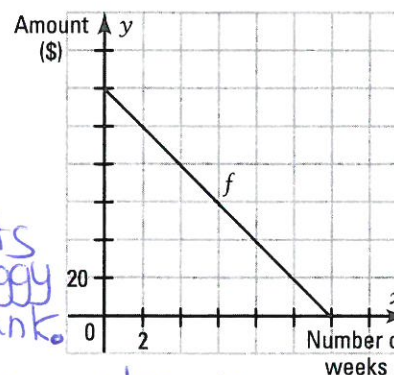
a) What is the domain of this function?

_____

b) What is the range of this function?

_____
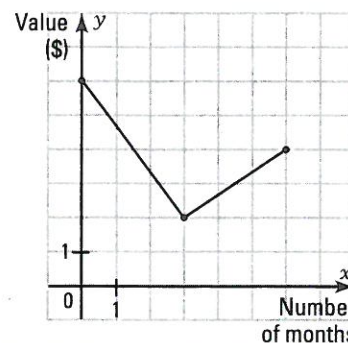
Activity 3 Initial value and zero of a function

Chloe takes $10 out of her piggy bank each week. The function f, represented on the right, gives the amount y (in dollars) left in her piggy bank as a function of the number x of weeks. This function has the rule $y = 120 - 10x$.

1. The initial value of a function is the value of y when $x = 0$. Calculate and interpret the initial value of the function f.

   $120. The initial value represents the initial amount of money in her piggy bank.

2. The zero(s) of a function is (are) the value(s) of x for which $y = 0$. Calculate and interpret the zero of the function f.

   $120 - 10x = 0 \Rightarrow x = 12$. After 12 weeks, the piggy bank is empty

---

INITIAL VALUE AND ZERO OF A FUNCTION

- The initial value of a function $y = f(x)$ is the value of y when $x = 0$.
  To determine the initial value, we calculate $f(0)$.
- The zero(s) of a function $y = f(x)$ is (are) the value(s) of x when $y = 0$.
  To determine the zero(s), we solve the equation $f(x) = 0$.

Ex.: The fonction $f(x) = 60 - 0.12x$ gives the quantity $f(x)$ of gas (in l) remaining in the tank as a function of the distance x traveled (in km) by a car.
  1. Initial value of f: $f(0) = 60$.
     The tank initially contains 60 l.
  2. Zero of f: $60 - 0.12x = 0 \Leftrightarrow x = 500$.
   The tank is empty after 500 km.

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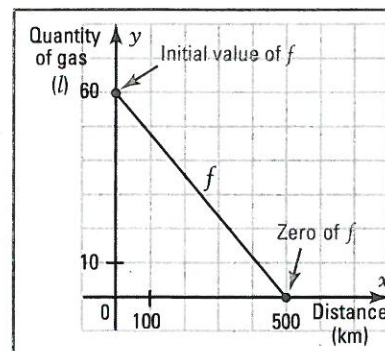
6. The graph on the right represents a function f which gives the profit p made by a company as a function of time (in months) since its opening.

a) What is the zero of this function? What does it represent?
3. It represents the number of months since opening for the profit to be zero.

b) What is the initial value of this function? What does it represent?
_At its opening, the company registers a loss of $1500._
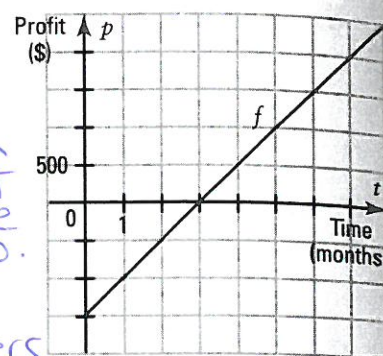
7. The graph on the right represents a function f which gives the height h reached by a diver (in metres) as a function of time (in seconds).

a) What are the zeros of this function? What do they represent?
1s and 4s. They represent the time at which the diver was at the surface of the water.

b) What is the initial value of this function? What does it represent?
4m. It represents the height from which the diver jumped
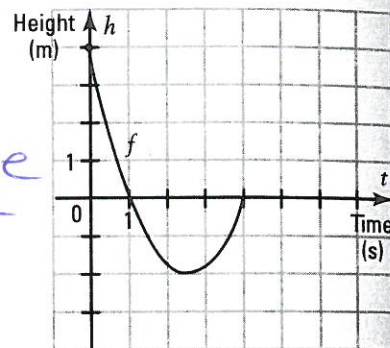
ACTIVITY 4 Sign of a function

The graph on the right illustrates the function which gives the net revenue of a travel agency as a function of the number of tourists taking a guided tour.

a) What is the zero of the function? What does it represent?
8. The profit is zero if 8 tourists take the guided tour.

b) Over what interval must the number of tourists be for the travel agency to lose money? _Over the interval [0, 8[._

c) Over what interval must the number of tourists be for the travel agency to make a profit? _Over the interval]8, +∞[_
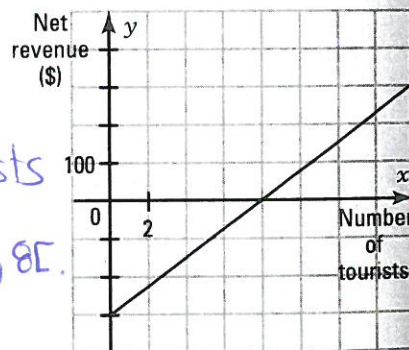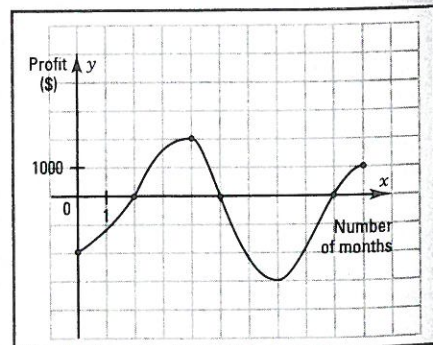

SIGN OF A FUNCTION

- Studying the sign of a function consists of finding the values of x for which the function is positive or those for which the function is negative.

    Ex.: The graph on the right associates, to the time x elapsed in months, the monthly profit y of a company during the first ten months of operation. This function has three zeros: 2, 5 and 9.
    The function is positive in $[2, 5] \cup [9, 10]$.
   The function is negative in $[0, 2] \cup [5, 9]$.

VARIATION OF A FUNCTION

- A function f is **constant** over an interval if, when x increases over that interval, the image $f(x)$ remains constant.
- A function f is **increasing** over an interval if, when x increases over that interval, the image $f(x)$ increases or remains constant.
- A function f is **decreasing** over an interval if, when x increases over that interval, the image $f(x)$ decreases or remains constant.

Ex.: The function f on the right associates, to the hour x of the day, the temperature y (in ° C) observed.

The function f is
- increasing in $[8, 12]$;
- constant in $[12, 13]$;
- decreasing in $[13, 16]$.
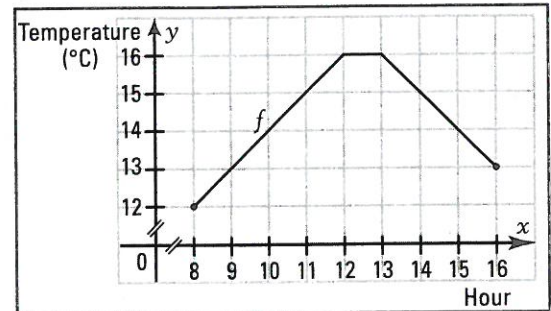
10. Liam blows into a balloon for a few seconds and then allows it to deflate. The graph on the right represents the function f which associates, to the time t in seconds elapsed from the beginning, the volume v in cubic centimeters of the balloon. Find and interpret the interval where the function f is:

a) increasing; _[0,8]. From 0 to 8 mins, the balloon is filling with air._

b) decreasing. _[8,12] From 8 to 12 mins, the air is let out from the balloon._
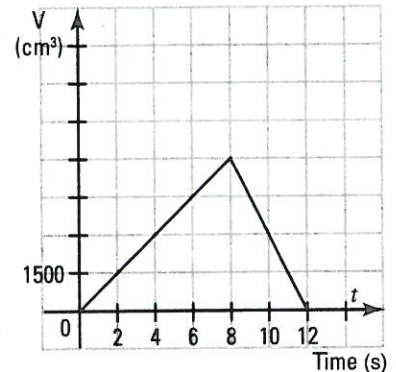
11. Raphael has taken a bath. The graph on the right illustrates the variation of the volume of water in the bath from the moment she turned on the faucet.

a) Indicate and interpret the interval where the function V is increasing.
[0,6]. From 0 to 6 minutes, Raphaelle is filling her bath.

b) Indicate and interpret the interval where the function V is constant.
[6,14]. It is over this interval of time, 8mins, that the Raphaelle bathe

c) Indicate and interpret the interval of time where the function V is strictly decreasing.
[14,18]. It is over this interval of time, 4mins, that the bath is emptied.
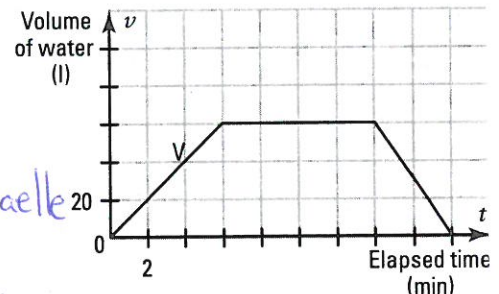
ACTIVITY 6 Extrema of a function

The graph on the right illustrates the flight of a kite as a function of the elapsed time since the kite's release.
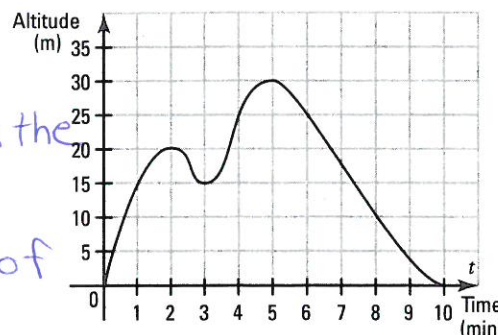
a) Determine and interpret the domain of the function.
dom A = [0,10]. The kite was in the air for 10 mins.

b) Determine at what instant the kite reaches its maximum altitude. What is this maximum altitude?
*It reaches a maximum altitude of 30 m after 5 mins.*

c) What is the kite's minimum altitude? At what time(s) is it reached?
The minimum altitude is 0m at the start (0min) and at its return 10 min later.

MAXIMUM AND MINIMUM OF A FUNCTION

- The **maximum** (**minimum**) of a function is the highest value (lowest value) of the dependent variable.

  Ex.: A projectile is launched from a height of 24 m above ground. The graph on the right represents the function f which associates, to the time t elapsed in seconds from the beginning, the height h in meters of the projectile.
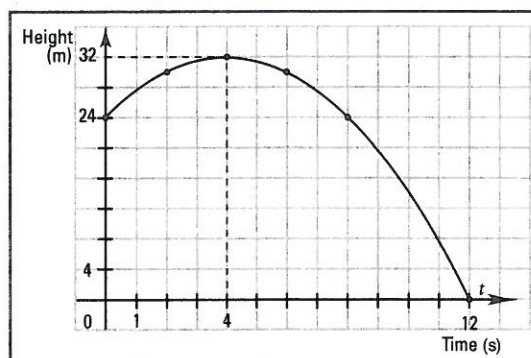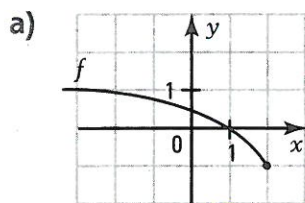
  – The maximum of the function f is 32. We note: max $f = 32$.
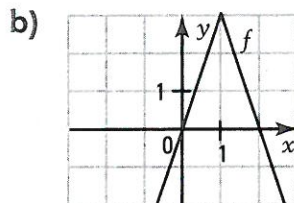 The projectile reaches a maximum height of 32 m at time $t = 4$ s.
  – The minimum of the function f is 0. We note: min $f = 0$.
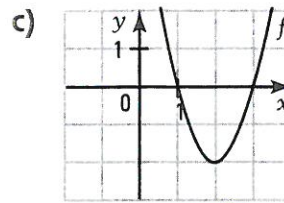 The projectile reaches a minimum height of 0 m at time $t = 12$ s.

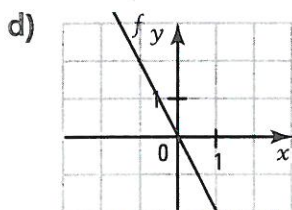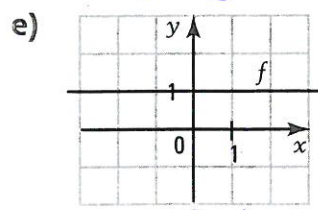**12.** Determine, when they exist, the maximum and minimum of the following functions.
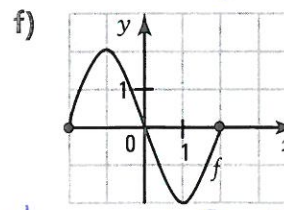
a)
min f = -1

b)
max f = 3

c)
min f = -2

d)
No extrema

e)
max f = 1, min f = 1

f)
max f = 2, min f = -2

1.1 Properties of functions