

► **Length of \overline{HC}**

$$(m \overline{AH})^2 = m \overline{HB} \times m \overline{HC}$$

$$(7.2)^2 = 5.4 \times m \overline{HC}$$

$$m \overline{HC} = 9.6 \text{ cm}$$

(In a right triangle, the height is the geometric mean of the lengths of the projections of its legs onto the hypotenuse.)

► **Length of \overline{BC}**

$$m \overline{BC} = m \overline{HB} + m \overline{HC}$$

$$m \overline{BC} = 5.4 + 9.6$$

$$m \overline{BC} = 15 \text{ cm}$$

► **Length of \overline{AC}**

$$(m \overline{AC})^2 = (m \overline{HA})^2 + (m \overline{HC})^2 \quad (\text{Pythagorean theorem in triangle AHC})$$

$$(m \overline{AC})^2 = (7.2)^2 + (9.6)^2$$

$$(m \overline{AC})^2 = 144$$

$$m \overline{AC} = 12 \text{ cm}$$

► **Perimeter of triangle ABC**

$$\begin{aligned} \text{Perimeter } \Delta ABC &= m \overline{AB} + m \overline{BC} + m \overline{AC} \\ &= 9 + 15 + 12 \\ &= 36 \text{ cm} \end{aligned}$$

► **CONCLUSION**

The perimeter of triangle ABC is equal to 36 cm.

13. AN ISOSCELES RIGHT TRIANGLE

► **Coordinates of point G**

$$C(1, 2); D(9, 8)$$

$$x_G = \frac{1}{2}(1 + 9) = 5 \text{ and } y_G = \frac{1}{2}(2 + 8) = 5$$

$$\text{We have: } G(5, 5)$$

► **Coordinates of point F**

$$A(9, 10); B(13, 2)$$

$$x_F = 9 + \frac{3}{4}(13 - 9) = 9 + \frac{3}{4}(4) = 12$$

$$y_F = 10 + \frac{3}{4}(2 - 10) = 10 + \frac{3}{4}(-8) = 4$$

$$\text{We have: } F(12, 4)$$

► **Classification of triangle EFG**

$$E(8, 1); F(12, 4); G(5, 5)$$

$$\bullet m \overline{EF} = \sqrt{(12 - 8)^2 + (4 - 1)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$$

$$m \overline{EG} = \sqrt{(5 - 8)^2 + (5 - 1)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

Since $m \overline{EF} = m \overline{EG}$, triangle EFG is isosceles with main vertex E.

$$\bullet \text{Slope } EF = \frac{4 - 1}{12 - 8} = \frac{3}{4}; \text{slope } EG = \frac{5 - 1}{5 - 8} = \frac{-4}{3}$$

Since slope of $EF \times$ slope of $EG = -1$, we have: $\overline{EF} \perp \overline{EG}$

Triangle EFG is a right triangle at E.

► **CONCLUSION**

Triangle EFG is an isosceles right triangle at E.

